The Geometry of Plato's Cosmos

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Abstract. Current understanding of astronomy attributes the earliest geometric models to the Greeks. Yet there remains substantial uncertainty about the Mesopotamian origins of the classical Greek constellations. It is here shown how clues famously given by Plato in his *Timaeus* provide the key to understanding the original geometric design framework. Having allocated the four elements (Water, Earth, Fire and Air) to regular polyhedra, Plato assigned a fifth figure to the cosmos, traditionally identified as the dodecahedron. Based on geometrical and philosophical arguments, it is here proposed that Plato also had in mind the orbicular elevated dodecahedron, consisting of 360 fundamental Platonic scalene triangles. In mapping it out as a convenient approximation to the celestial sphere, we discover that it offers a geometric framework for the Paths of Anu, Enlil and Ea of Mesopotamian astronomy, while explaining the enigma of why the constellations of the zodiac are not equally distributed along the ecliptic. Three rings with partial ten-way rotational symmetry are also identified that appear to have been used in the design framework. The conclusions emphasize Plato's debt to earlier astronomers, while transforming our understanding of the constellations so familiar today.

Introduction

The extent to which Plato's ideas were informed by earlier traditions has long been a matter of debate.[1] It was even reported in antiquity that Plato had obtained material for the *Timaeus* from an unpublished book by the

[1] For example, F.M. Cornford, *Plato's cosmology. The Timaeus of Plato* (London: Routledge, 1937, reprinted, Hackett, 1997) [hereafter Cornford], pp.i–x, 1–8; J.N. Findlay, *Plato. The written and unwritten doctrines* (London: Routledge & Kegan Paul Ltd, 1974), pp.23, 59; P.S. Horky, *Plato and Pythagoreanism* (Oxford University Press, 2013) [hereafter Horky *P&P*], p.215; A. Uždavinys, *Philosophy & theurgy in late antiquity* (Kettering, OH: Angelica Press / Sophia Perennis, 2014) [hereafter Uždavinys], pp.9–21, 70–73.

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Pythagorean philosopher Philolaus.[2] Whether or not that is true, the historian Neanthes also noted how Plato was excommunicated from the Pythagorean community for publishing their secrets too openly.[3]

Pythagorean learning was famous in antiquity for being a closely guarded secret, leading to the ancient suggestion that Plato's writings represent a giving away of reserved knowledge, drawing especially on Egyptian and Mesopotamian traditions.[4] Several of Plato's own writings have often been seen as not being entirely open, as illustrated by Plato's *Phaedrus* and as reflected by commentators from Plutarch to Ficino.[5] By contrast, modern scholarship has often tended to emphasize innovation by Plato and to distance itself from associating Plato with antecedent sources, including any mystery tradition.[6] For example, a famous fragment attributed to Philolaus already links the physical elements to geometry: 'And in the sphere there are five bodies: those in the sphere, fire, water,

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[2] Diogenes Laertius, *Lives of Eminent Philosophers*, trans. R.D. Hicks (London: Heinemann, 1925) [hereafter DL, Hicks], 8.85, Vol. 2, p.399; W. Burkert, *Lore and science in ancient Pythagoreanism*, trans. E.L. Minar, Jr (Cambridge, MA.: Harvard University Press, 1972) [hereafter Burkert *Lore & Science*], pp.224–229.

[3] DL 8.55 (Hicks, p.371).

[4] For example, Pliny, *Natural History*, trans. J. Bostock and H.T. Riley (London: Bohn, 1855) [hereafter Pliny, *NH*, Bostock & Riley] 30.2, Vol. 5, pp 424–425; Iamblichus, *On the Mysteries & Life of Pythagoras*, trans. T. Taylor (originally, 1818; republished Sturminster Newton: Prometheus Trust, 2004) [hereafter Iamblichus, *Pythagorean Life*, Taylor], p.268); Burkert, *Lore & Science*, pp.224–229.

[5] Plato, *Phaedrus* 246A–250C, 274B–277A; cf. *Timaeus* 48B, reading *oudeis memēnuken* as 'no one revealed'; Plutarch, *Moralia*, *Vol. V*, trans. F.C. Babbit (Cambridge, MA.: Harvard University Press, 1936) [hereafter, Plutarch, Babbit], *On Isis and Osiris* 370F, 382D,E (pp.119, 181–183); Marsilio Ficino, *All Things Natural. Ficino on Plato's Timaeus,* trans. A. Farndell (London: Shepheard-Walwyn (Publishers) Ltd., 2010). *Compendium in Timaeum* 43, pp.95–96.

[6] For example, E. Sachs, 'Die fünf platonischen Körper. Zur Geschichte der Mathematik und der Elementenlehre Platons und der Pythagoreer', *Philologische Untersuchungen* 24. Heft (Berlin: Weidmannsche Buchhandlung 1917; republished Hamburg: Severus Verlag, 2010) [hereafter Sachs], p.7: 'The cloud of Pythagorean mysticism that rests over the Timaeus is the work of the successors of Plato, started by Speusippus, Xenocrates and Philip. They are to blame for multiple misunderstandings and some mistakes, they reinterpreted science in mysticism, and in the myth they saw the revelation of the truth' (trans.). cf. A. Gregory, 'Mathematics and cosmology in Plato's *Timaeus*', *Apeiron* (March 2021), p.11, https://doi.org/10.1515/apeiron-2020-0034

earth, and air, and fifthly the cargo-ship of the sphere'.[7] Guthrie thought that the fragment could well be authentic, considering it likely that Plato was 'adopting and elaborating Pythagorean notions'. By contrast, Burkert and Huffman rejected the fragment as spurious, at the heart of which seems to be their concern that it is too characteristically Platonic.[8] It is not proposed here to resolve such differences of opinion, but rather to recognize that, to some greater or lesser extent, Plato must have depended on predecessors and that it becomes difficult to know exactly how much he innovated or inherited.

The uncertainties are even larger when it comes identifying Plato's sources. Scholars have questioned the authenticity of almost every claim attributed to Pythagoras and the Pythagoreans.[9] It has been suggested that the Pythagoreans may have used only arithmetical approaches, while geometrical approaches were a later Greek invention.[10] By contrast, drawing on architectural and engineering evidence, it has been argued that

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[7] Philolaus, Fragment 12, reading *holkas* as a sailing ship; cf. W.K.C. Guthrie, *A history of Greek Philosophy, Volume 1, The Earlier Presocratics and the Pythagoreans* (Cambridge: Cambridge University Press, 1962) [hereafter Guthrie], pp.267–268; C.A. Huffman, *Philolaus of Croton. Pythagorean and Presocratic* (trans. with commentary) (Cambridge: Cambridge University Press, 1993) [hereafter Huffman, *Philolaus*], p.393.

[8] Guthrie, p. 268; Burkert, *Lore & Science*, p. 276; Huffman, *Philolaus*, pp. 392–395; cf. G. Lloyd, *Pythagoras*, in C. Huffman, ed., *A History of Pythagoreanism* (Cambridge: Cambridge University Press, 2014), pp.25–27, 45.

[9] For example, L. Zhmud, *Pythagoras and the early Pythagoreans* (Oxford: Oxford University Press, 2012) [hereafter Zhmud, *Pythagoras*]; Zhmud, Early Mathematics and Astronomy. Chapter B2, in P.T. Keyser and J. Scarborough, eds, *Oxford Handbook of Science and Medicine in the Classical World* (Oxford: Oxford University Press, 2018) [hereafter Zhmud, *Early M&A*], pp.176, 190; R. Netz, The Pythagoreans, in T. Koetsier and L. Bergmans, eds, *Mathematics and the Divine: A historical study* (Elsevier B.V., 2005), pp.77–97; Horky, *P&P*, pp.3–30; C. Huffman, *Pythagoras* (Stanford Encyclopaedia of Philosophy, 2018) [hereafter Huffman, *Pythagoras*] https://plato.stanford.edu/entries/pythagoras/ [accessed 4 January 2021].

[10] A. Gregory, 'The Pythagoreans: number and numerology', in M. McCartney and S. Lawrence, eds, *Mathematicians and their Gods: Interactions between mathematics and religious beliefs.* (Oxford: Oxford University Press, 2015), pp.21–50 [hereafter Gregory, *Pythagoreans*] (see pp.29, 42); R. Netz, The problem of Pythagorean mathematics, in C. Huffman, ed., *A History of Pythagoreanism* (Cambridge: Cambridge University Press, 2014), pp.167–184 [hereafter Netz, *Problem*], see p.181; see Supplementary Discussion S3.5.

the Egyptians and Babylonians already knew of geometrical approaches, which were inherited by Thales, Pythagoras and others.[11]

The scholarly evaluation seems to have been overly dependent on individual fragments, such as an undated scholion to Euclid's *Elements* 13.1, reporting that, whereas the cube, tetrahedron and dodecahedron are from the Pythagoreans, the octahedron and icosahedron are of Theaetetus, an associate of Plato.[12] Yet scholars have hardly settled precisely what this scholion meant, whether discovery, geometric construction, inscribing within a sphere, or the written text itself, including Euclid's associated proofs.[13] What appears to survive the wider scholarly criticism as authentic is that certain teachings of the Pythagoreans were secret and that the Pythagoreans had significant interest in number symbolism.[14] While the limited extent of early textual sources makes such debates inevitable, this uncertainty is less critical for the present purpose than the question of dependence by Plato on his predecessors, whoever they may have been.

Such a view of Plato as an inheritor of prior knowledge is not to denigrate Plato's intellectual contribution to developing ideas current at the time, but rather to place his writings in their proper context. We might make parallels with Robert Boyle, with both figures straddling periods of rapidly increasing scientific openness, while showing affinities to both the

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[11] R. Hahn, *The metaphysics of the Pythagorean theorem. Thales, Pythagoras, engineering, diagrams, and the construction of the cosmos out of right triangles* (Albany, NY: State University of New York Press, 2017) [hereafter Hahn, *Metaphysics*], pp.10, 41–43, 243; cf. L. Zhmud, Sixth-, fifth- and fourth-century Pythagoreans, in C. Huffman, ed., *A History of Pythagoreanism* (Cambridge: Cambridge University Press, 2014), pp.88–111 [hereafter Zhmud, *Sixth–Fourth*] (see pp.95, 105).

[12] Euclid, *Euclidis Elementa*, ed., I.L. Heiberg (Lipsiae: B.G. Teubneri, 1888), Vol. 5, p.654; for trans. see Supplementary Section S3.7.2.

[13] W.C. Waterhouse, 'The discovery of the regular solids', *Archive for the History of Exact Sciences* 9 (1972): pp.212–221 [hereafter Waterhouse] (see pp.212–213); Hahn, *Metaphysics*, pp.201–204; cf. Sachs, p. 29 ff; J. Burnet, *Early Greek Philosophy*, 3rd edn (London: A. & C. Black Ltd, 1920), p.284; C. Huffman, 'The peripatetics on the Pythagoreans', in C. Huffman, ed., *A history of Pythagoreanism* (Cambridge: Cambridge University Press), pp.274–295 (see pp.278–280); J. Palmer, 'The Pythagoreans and Plato', in C. Huffman, ed., *A History of Pythagoreanism* (Cambridge, Cambridge University Press, 2014), pp.204–226 (see p.223); Zhmud, *Sixth–Fourth*, p.105; Zhmud, *Early M&A*, pp.189–190.

[14] P.T. Struck, *Birth of the Symbol. Ancient readers at the limits of their texts* (Princeton, NJ: Princeton University Press, 2004) [hereafter Struck], pp.96–104; Gregory, *Pythagoreans*, pp. 32–35; Huffman, *Pythagoras*, sections 4.3, 5.

old traditions and the new rationalism.[15] In this way, Plato's supposed revealing of secret knowledge would appear not to have been a simple black-and-white process, but rather a matter of gradual explanation and cryptic hints, encouraging readers to think through the logical next steps for themselves.[16] A similar approach with intellectual gaming may also have been embraced by Plutarch in his discussion of Plato's geometry[17].

It is with this perspective that I suggest we should reconsider Plato's famous description of the cosmos, which is made up of the four elements: water, earth, fire and air (*Timaeus*, 53C–55C). The concept of these elements was taken over from Philolaus' near contemporary, the philosopher Empedocles – neither of whom Plato actually mentions in the *Timaeus*. Whatever he owed to such prior sources, Plato's description of the elements as being made up of component triangles is the earliest known account of this theory. Plato's own position as someone selectively sharing some information, while retaining other points deliberately unexplained, is perhaps most clearly seen in his hint about the fifth shape: 'There was yet one more construction, the fifth, God used it on the All for its decoration' (*eti de ousēs xustaseōs mias pemtēs, epi to pan ho theos autē katechrēsato ekeino diazōgraphōn*, *Timaeus* 55C). It is worth noting that Plato did not explicitly mention that he was referring to the dodecahedron, although this is the only regular convex polyhedron remaining. The inference may, however, also be made from his *Phaedo,* where Plato likens the true world to a ball stitched together from twelve pieces of leather.[18] In both accounts, Plato has deliberately chosen to let the reader infer that the dodecahedron is meant, while carefully avoiding to name or describe it exactly.

While Plato's account hinting about the dodecahedron at *Timaeus* 55C is brief, it is also pregnant with meaning. For example, Waterfield renders

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[15] R. Janko, 'The Derveni papyrus (Diagoras of Melos, *apopyrgizontes logoi*?): A new translation', *Classical Philology* 96 (2001): pp.1–32 [hereafter Janko, *Derveni papyrus*]; L.M. Principe, 'Robert Boyle's alchemical secrecy: Codes, cyphers and concealments', *Ambix* 39 (1992), pp.64–74.

[16] Taylor, Introduction to Iamblichus, *Pythagorean Life*, Vol. 1, pp.3–4; A. Gregory, Introduction and Notes, in Plato, *Timaeus and Critias*, trans. R. Waterfield (Oxford: Oxford University Press, 2008) [hereafter Gregory, Notes on Plato's *Timaeus*], p.xl.

[17] Plutarch, *The Obsolescence of Oracles* 428A,B (Babbit, p. 447); J. Opsomer, Plutarch on the geometry of the elements, in M. Meeusen and L. Van der Stockt, eds, *Natural Spectaculars: Aspects of Plutarch's Philosophy of Nature* (Leuven: Leuven University Press, 2015), pp.29–55 [hereafter Opsomer] (see pp.42–43).

[18] Plato, *Phaedo* 109A-110D, in Plato, *The Last Days of Socrates*, trans. H. Tredennic (Harmondsworth: Penguin, 1958), pp.146–147.

the line as: 'There remained one further construct, the fifth; the god decorated it all over and used it for the whole'. This needs careful reading to notice that the 'all over' refers to *epi to pan*, since Plato had specified that the fifth construction was used 'on the All'. By comparison, Bury's translation completely misses this allusion: 'And seeing that there still remained one other compound figure, the fifth, God used it up for the Universe in his decoration thereof'.[19] The other keyword to notice is *diazōgraphōn,* translated here as 'decoration'. As Bury, Cornford and Gregory noticed, this may have been an allusion to the zodiac, where *diazōgraphōn* could also be taken literally as 'through animal drawing'.[20]

The most celebrated application of the Platonic Solids in recent centuries is surely that by Johannes Kepler, who showed that the spacing of these polyhedra nested within each other could explain the distances between the orbits of the known planets of the solar system.[21] Like Ficino and other earlier writers, Kepler also thought it likely that Plato's obscurity reflected a deliberate hiding of reserved knowledge about these polyhedra.[22] However, given the range of scholarly opinion over the last century, it seems unlikely that a sole dependence on well-studied texts could bring any resolution to such questions, including whether the likes of Plutarch, Ficino and Kepler were in any way right about Plato's crafted obscurity.[23]

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[19] Plato, *Timaeus* 55c, in Plato, *Timaeus, Critias, Cleitophon, Menexenus, Epistles*, ed. and trans. R.G. Bury, (Cambridge, MA.: Harvard University Press, 1929) [hereafter, Plato, *Timaeus*, Bury], p.135; Plato, *Timaeus and Critias*, trans. R. Waterfield (Oxford: Oxford University Press, 2008) [hereafter Plato, *Timaeus*, Waterfield], p.49.

[20] Bury, Notes to Plato, *Timaeus*, p.134; Cornford, p.218; Gregory, Notes on Plato's *Timaeus*, p.144. For arguments against this interpretation, see R. Kotrč, 'The dodecahedron in Plato's *Timaeus*', *Rheinisches Museum für Philologie*, Vol. 124 (1981), pp.212–222.

[21] J. Kepler, *Mysterium Cosmographicum. The Secret of the Universe*, trans. A.M. Duncan, (New York: Abaris, 1981) (reprint of the 2nd edition of 1621) [hereafter Kepler, *Mysterium Cosmographicum*, 1981], p.155; J.V. Field, 'Kepler's cosmological theories: their agreement with observation', *Quarterly Journal of the Royal Astronomical Society* 23 (1982): pp.556–568 [hereafter Field, *Kepler's cosmological theories*].

[22] Kepler, *Harmonicies Mundi*, Libri V., Book II. Section xxv, trans. in J.V. Field, 'Kepler's star polyhedra', *Vistas in Astronomy* 23 (1979): pp.109–141 [hereafter Kepler, *Harmonicies Mundi*, Field] (see p.124).

[23] Sachs, p.7; Guthrie, p.267; E.G. MacClain, *The Pythagorean Plato: Prelude to the song itself* (Boulder, CO.: Nicolas Hays, 1978), pp.143, 153; P.S. Horky,

The present study applies a fresh approach, identifying and exploring an alternative geometric model of Plato's dodecahedral cosmos, then testing it empirically in relation to evidence from cosmic geography and astronomy. As far as I can see, the dodecahedral model substantially pre-dates Plato. While acknowledging the wide range of scholarly views, the examination here offers independent evidence for Plato as an inheritor of a prior dodecahedral tradition for which we have scant written evidence.[24] As the conclusions are likely to be contentious, I apply a set of nine evaluation criteria (Supplementary Methods S1.3) to assess the results. These criteria are helpful in informing: a) the extent to which correspondences are significant/meaningful (rather than simply chance coincidence) and b) what such correspondences reveal about ancient ideas. It is hoped that the approach may encourage wider use of such criteria across the history of science.

In the end, I find myself agreeing with Kepler that Plato's geometrical account both hints and hides. The results also suggest that Kepler missed a key feature of Plato's system.

1. Theoretical and historical foundations

My starting point was that, according to Plato's explanation, the dodecahedron remains something of an impossibility. The polyhedra for Water, Air and Fire are described by Plato as constructed from equilateral triangles, each of which is composed of six primary scalene triangles (sides of length 1:√3:2) (*Timaeus* 53C–57D). In the case of Earth, the square sides are each composed of four isosceles triangles (sides of length 1:1:√2)[25] – see Figure 1. If the dodecahedron represents the All, then prior thinking led me to expect that it should be inter-convertible in some way with the other polyhedra by rearrangement of these scalene or isosceles triangles.[26] For the simple dodecahedron, however, it is not possible to use Plato's primary triangles, since each face is composed of five isosceles triangles with sides 1:1:√~1.4.

'Persian cosmos and Greek philosophy: Plato's associates and the Zoroastrian magoi', *Oxford Studies in Ancient Philosophy* 37 (2009): pp.47–103 (see p. 66 ff.); Zhmud, *Pythagoras*, pp.239–281; Uždavinys, pp.9–21, 70–73.

[24] 'The elevated dodecahedron as a model of the cosmos: Evidence from written sources' (forthcoming) [hereafter *Elevated dodecahedron*, forthcoming].

[25] Cornford, pp.210–239.

[26] See Supplementary Discussion S3.1 for further arguments. I later noted that this could also be concluded from the *Timaeus* 32B,C, where Plato notes that the heavens were joined together and constructed from all of the elements.

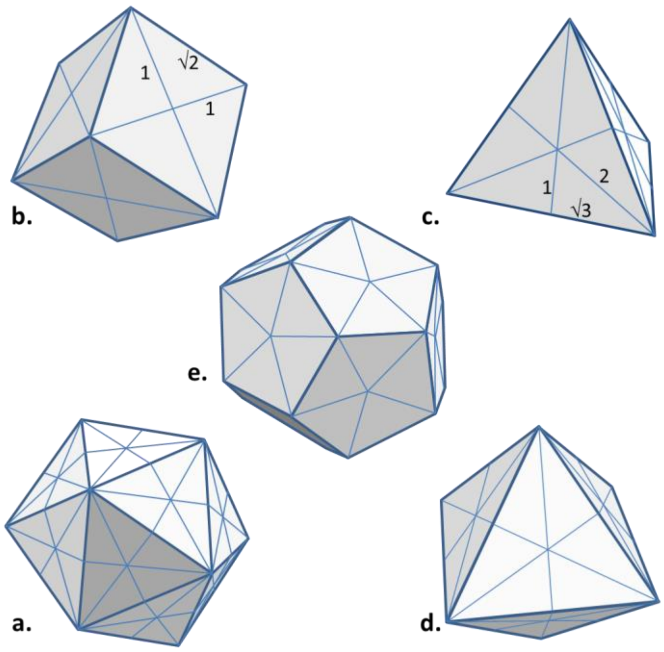

Figure 1: The Platonic Solids, the regular polyhedra with which Plato identified the four classical elements: a. icosahedron (Water); b. cube (Earth); c. octahedron (Air); and d. tetrahedron (Fire). He then mentioned a fifth figure to represent the whole cosmos, which has traditionally been identified with the dodecahedron (e). Plato described the polyhedra for Water, Air and Fire as being formed from fundamental scalene triangles (sides 1:2:√3), with the cube composed from isosceles triangles (sides 1:1:√2). (Images © Mark Sutton)

At this point, it is worth noting comments by both Plutarch and Proclus which can be interpreted as suggesting that Plato's writing about a fifth figure showed that he was aware of the existence of aether as the fifth essence ('quintessence'), which Plato and Aristotle elsewhere explained as making the substance of heaven.[27] To this we should recall a common view

[27] Plutarch, *The E at Delphi* 389F–390A (Babbit, pp.227–229); Proclus, *Commentary on the Timaeus of Plato.* (2 vols.), trans. T. Taylor (1820, reprinted, Sturminster Newton: The Prometheus Trust, 2005) [hereafter Proclus, *In Timaeum*, Taylor], 1.1.6, 63 (Vol. 1, pp.16, 66); cf. Plato, *Phaedo* 109B,C; *Epinomis* 981C; Aristotle, *De Caelo*, 270C; Supplementary Discussion S3.1,

from antiquity that the Earth is but a point compared with the size of the Heavens, implying that most of the cosmos should consist of aether.[28] While acknowledging that it remains contentious whether Plato ever actually associated aether with the dodecahedron, if we should choose, it therefore appears more likely that Plato's dodecahedral All would be made up of scalene triangles (sides 1:√3:2), rather than the isosceles triangles of Earth, especially as aether constitutes the ever-living divine fire, perhaps made from the best parts or 'summits' of the elements.[29]

Based on these initial considerations, the obvious next step was to see how the dodecahedron would appear when its faces are elevated by combining five equilateral triangles, since these are composed from Plato's fundamental scalene triangles. Only much later did I discover that Plutarch had also described this construction (*Platonic Questions* V, 1003D), concerning which Sachs and Cherniss had simply assumed that Plutarch had been mistaken.[30] In this way, each face of the dodecahedron appears as a blunt pentagonal pyramid.[31] Although this division may also be done with the pentagonal pyramids pointing inwards, I chose the externally pointing orientation, since this would be 'most like the sphere', thereby constructing an elevated dodecahedron (Figure 2a,b).[32]

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recognizing doubts about the reliability of late Neoplatonic sources such as Proclus.

[28] Macrobius, *Commentary on the Dream of Scipio*, trans. W.H. Stahl (New York: Columbia University Press, 1952, reprinted 1990) [hereafter Macrobius, *Dream*, Stahl], I.16.10, II.5.10–II.9.9 (see pp.154 and notes, 201–215); Martianus Capella, *Martianus Capella and the Seven Liberal Arts. Vol. II. 'The Marriage of Philology and Mercury'*, trans. W.H. Stahl, R. Johnson and E.L. Burge (New York: Columbia University Press, 1977) [hereafter Martianus Capella, *Marriage*, Stahl *et al.*] VI.583, p.219.

[29] Plato, *Phaedo* 109B–110A; Proclus, *In Timaeum* 1.2.42–2.58 (Taylor, Vol. 1, pp.457–471), commenting on *Timaeus* 32B,C.

[30] Sachs, pp.15–17; H. Cherniss, Notes to Plutarch, *Moralia, Vol. XIII, Part. 1*, trans. H. Cherniss (Cambridge, MA.: Harvard University Press, 1976) [hereafter Plutarch, Cherniss], p.54; cf. *Elevated dodecahedron*, forthcoming.

[31] cf. the *koruphai*, the heads or tips of the icosahedron, Sachs, p.13.

[32] Timaios of Locri, *On the Nature of the World and the Soul*, ed. and trans. T.H. Tobin (Chico, CA: Scholars Press, 1985) 98D, pp.51, 76.

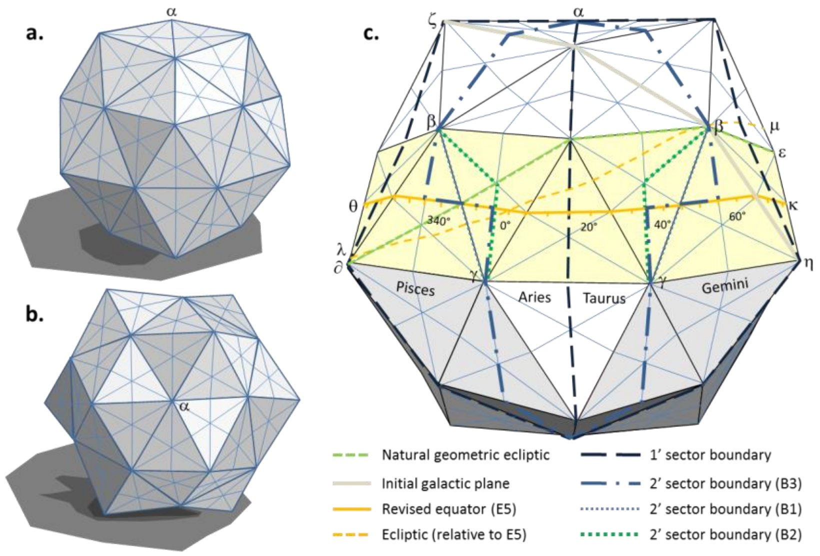

Figure 2: Three views of the orbicular elevated dodecahedron, showing its composition from equilateral and scalene triangles (sides 1:2:√3). a. View centred on a pentagonal pyramid; b. view centred on a hexagonal surface; c. view showing the primary (1') and secondary (2') boundaries between zodiac constellation sectors showing three variants of the 2' boundary (B1, B2 and B3, β–γ), initial estimates of the Galactic Plane (ζ–η) and natural geometric ecliptic (∂–ε), with equator E5 (θ–κ, c.f. Supplementary Figure 2) and actual ecliptic (λ–μ), where 0° Right Ascension (RA) is here plotted as the mid-point of the 30-day zodiacal month of Aries (Nisannu). (Images © Mark Sutton)

The first publication explicitly naming this polyhedron (*dodecaedron elevatum*) was by Luca Pacioli in 1509, using woodcuts for which he commissioned drawings by Leonardo da Vinci.[33] However, Pacioli's terminology does not specify any particular degree of elevation. To refer specifically to the form using equilateral triangles, I therefore extend this as *dodecaedron elevatum orbicularis* or 'orbicular elevated dodecahedron' (OED), which reflects both the sphere-like and cosmological import of this polyhedron.[34]

[33] L. Pacioli, *De Divina Proportione* (Venice: A. Paganius Paganinus, 1509) [hereafter Pacioli, *Divina Proportione*], p.16, plates XXXI, XXXII.
[34] cf. Macrobius, *Dream* I.14.24–25 (Stahl, p.148).

Although Pacioli entitled his book the *De Divina Proportione*, he gave no clear account of cosmic associations for the OED. The earliest ‘net’ or unwrapped outline of the OED was subsequently published by Daniel Barbaro, accompanied by a perspective view with more-pointed pyramids.[35] Kepler himself was later the first to describe the related ‘small stellated dodecahedron’ (SSD), which he termed *echinus* (hedgehog or urchin), where the triangular faces (1:1:√~0.4) surrounding each pyramid form the planes of twelve pentagrams. Although Kepler linked the SSD to his planetary cosmology, he appears to have made no mention of the OED.[36]

My initial satisfaction with the OED focused on the way that it extended the numerical resonances of the All that would have been so appreciated by the Pythagoreans and others interested in number symbolism (see Introduction). Anticipating that they would have found this polyhedron of interest, this gave me confidence to examine the OED in more detail. The regular dodecahedron already contains a five-way symmetry in the sides of each face (i.e., all the five fingers). It also combines twelve faces (all the months of a year) with thirty edges (all the days of a month). However, the OED extends these to include the sixty faces of the equilateral triangles, representing the All of the Mesopotamian sexagesimal system. Finally, the OED is composed of 360 fundamental scalene triangles, representing all the degrees in a circle and all the days in the ‘ideal year’ of ancient Mesopotamian astronomy.[37] Again, it was only later that I found such expected resonances attested by both Plutarch and Alcinous.[38]

In this way, the OED provides an answer to the puzzle of why Plato had described the equilateral triangle as being composed from six small rather than two large fundamental scalene triangles of the same shape, since the latter would not have generated the All of 360, nor fully exploited the

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35 D. Barbaro, *La practica della perspective*, (Venice: Camillo & Rutilio, 1567), p.106.

36 Kepler, *Harmonices Mundi*, Books II, V. (Field, pp.110, 128). See Supplementary Discussion S3.1.

37 H. Hunger and D. Pingree, ‘MUL.APIN An astronomical compendium in cuneiform’, *Archiv für Orientforschung* Beiheft 24 (Horn, Austria: F. Berger & Söhne, 1989) [hereafter Hunger & Pingree, MUL.APIN], pp.139, 143, 145; L. Brack-Bernsen, ‘The path of the moon, the rising points of the sun, and the oblique great circle on the celestial sphere’, *Centaurus* 45 (2003): pp.16–31 [hereafter Brack-Bernsen] (see pp.23–26).

38 Plutarch, *Platonic Questions* 1003d (Cherniss, pp.53–55); Alcinous, *Handbook of Platonism* 13.2.

patterns of the OED (see below).[39] The OED is similarly curious in combining the (partial-) symmetry of both five or ten and six or twelve, depending on the angle of view (Figure 2a,b), especially given the nature of five and six as the roots of both circular and spherical numbers (25, 36, 125, 216). The benefit for increased symmetry of using six rather than two fundamental scalene triangles has also been noted.[40] We thus have a shape that is similar to a sphere, solves the puzzle of how to form a dodecahedron from Plato's fundamental triangles, and strongly characterizes the idea of the All, both through its numerical characteristics and its multiple symmetry.

2. Outlining the geometric cosmos

In reflecting on this shape, I realized that Plato's geometrical linking of elements and the cosmos assumed a correspondence between micro- and macro-scales (cf. *Timaeus* 27C–69A, 69A–92B, 92C). The implication was that the OED geometry could also be the basis for ordering the heavens, as seen in their apparent annual rotation through twelve months.

My next step was to locate the north and south celestial poles. It seemed obvious that someone wanting to divide the OED into twelve months would select point α in Figure 2 as the north celestial pole, and the point opposite as the south celestial pole. This allowed an easy primary division of the heavens into six two-month segments of 60°. However, while the boundary lines at each pole also divided conveniently into twelve, the joining up at the celestial equator of the remaining six secondary lines proved problematic, as these lines do not meet directly. As a first approximation, I therefore joined these at the equator in a balanced fashion (2' boundary lines B1: β–γ of Figure 2c).

In my initial analysis, I was excited to see that division of scalene triangles in the OED also offered a first approximation to the ecliptic, the annual path of the sun through the heavens, which I term the 'natural geometric ecliptic' (NGE, line ∂–ε, Figure 2c). Similarly, another line gave an initial estimate of the galactic plane, the central path of the Milky Way (line ζ–η). As I proceeded, my challenge was to find out how to map the OED in a way that would have appealed to the ancient astronomers: a) as exact as possible and b) following natural divisions of the OED. Through

[39] Cornford, pp.210–239; Tobin, p.76, notes to Timaios of Locri 98D.
[40] D.R. Lloyd, 'The chemistry of Platonic triangles: problems in the interpretation of the Timaeus', *HYLE – International Journal for Philosophical Chemistry* 13 (2007): pp.99–118.

several stages[41], I first identified that the equatorial zone could be easily divided into 36 decans each consisting of 10°, as well as into 24 hours (reflecting the apparent nightly rotation of the heavens). This provided a convenient basis for recording Right Ascension (RA) (division of the sphere around the celestial equator), giving a natural division to 5° RA (i.e., 5 days in the year or 20 minutes in the nightly rotation), with further division possible.

Locating the exact position of the celestial equator proved more challenging, since there are several potential ways to divide the scalene triangles to measure declination (δ) – the angle in degrees north and south of the equator. Of five alternatives, I eventually selected line θ–κ in Figure 2c based on four criteria (equator E5 of Supplementary Figure SF1; cf. Supplementary Table ST1). In particular, E5 provided a solution that was most consistent with the description of the actual ecliptic (i.e., line λ–μ) through the 'Path of Anu' as described in the Mesopotamian astronomical compendium MUL.APIN (see Supplementary Table 1). This survives in numerous cuneiform tablets (from c. 687 BCE), and appears to be based on astronomical observations from around 1200–1000 BCE or even earlier.[42] Based on MUL.APIN, the Path of Anu is thought to represent a band c. -17° to +17° δ around the celestial equator. According to recent scholarship, however, this band was not based on geometric models of the cosmos,[43] but was a region defined by arcs along the eastern horizon at the annual time of each star's first pre-dawn appearance (i.e., 'visible morning rising', VMR, or 'heliacal rising').[44] The sky to the north was known as the Path of Enlil, while to the south was the Path of Ea.

[41] As detailed in the Supplementary Methods, Section S1.1.
[42] Hunger & Pingree, introduction to MUL.APIN, pp.10–12; cf. The revised edition and trans. by H. Hunger and J. Steele, *The Babylonian astronomical compendium MUL.APIN* (Abingdon: Routledge, 2019) [hereafter Hunger & Steele, MUL.APIN]. These authors concur with the date of c. 1200–1000 BCE, but suggest greater uncertainty, c. ±450 years.
[43] As was later Greek cosmology and astronomy from around the 5th–4th century BCE, as illustrated by *Timaeus* 34C–36C and the *Phaenomena* of Eudoxus – see Supplementary Discussion S3.5.
[44] Brack-Bernsen, pp.23–26; J.M. Steele, 'Celestial measurement in Babylonian astronomy', *Annals of Science* 64 (2007): pp.293–325; J.M. Steele, *A brief introduction to astronomy in the Middle East* (London: Saqi, 2008), p.56; cf. Zhmud, *Pythagoras*, pp.317–321; Hunger & Steele, notes on MUL.APIN, p.171.

If the full range of δ from the celestial equator to each pole is divided into 15 equal units of 6° according to the natural OED divisions, then it can be seen that the band of 18 equilateral triangles around the equator of the OED provides a close estimate for the Path of Anu, representing δ -18° to +18°. Similarly, the Path of Enlil corresponds neatly on the OED to δ +18° to +54°, where the upper limit represents the edge of the ancient Arctic zone (assuming that observations were made from or intended to represent an 'ideal latitude' of 36° N, see further below). Accordingly, the Path of Ea corresponds on the OED to δ -18° to -54°, with the lower limit representing the edge of the celestial Antarctic zone. Contrary to recent opinion, we should therefore take seriously a hypothesis that the Mesopotamian astronomers adopted a geometric model (i.e., the OED) from which to establish the Paths of Anu, Enlil and Ea. As will be seen, it turns out that there is other evidence to support this hypothesis.

3. Explaining the zodiac enigma

While I was rather satisfied with these connections, the most surprising feature of the OED turned out to be its ability to explain the unequal lengths along the ecliptic of the twelve zodiac constellations. It has long been recognized that these constellations are of uneven length. For example, Virgo and Pisces are more than twice the length of Libra and Cancer.

Gurshtein attempted to explain these differences by suggesting that the first zodiacal constellations to be identified were those associated with the summer and winter solstices and the spring and autumn equinoxes.[45] In this way, he noted that the four constellations identified first, which he termed the 'Gemini Quartet' (Virgo, Pisces, Gemini, Sagittarius, respectively), had the largest area, fitting with a date of around 5600±150 BCE. As precession (the slow apparent cyclic movement of the heavens due 'wobble' of the earth's axis) gradually altered the observed location of the solstices and equinoxes, there was less space left, so that the subsequently named constellations were smaller (e.g., Leo, Aquarius, Taurus, Scorpio, respectively, by 2700±250 BCE). However, this explanation requires extremely ancient dating (which is not otherwise supported) and only

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[45] A. Gurshtein, 'When the zodiac climbed into the sky', *Sky & Telescope* (October 1995: pp.28–33. On the Babylonian origins of the zodiac, cf. C. Mitchell, 'Did the division of the year by the Babylonians into twelve months lead to adoption of an equal twelve-sign zodiac in Hellenistic astrology?' (Dissertation, Bath Spa University College, UK, 2008), pp.47–50; J. Steele, 'The development of the Babylonian zodiac', *Mediterranean Archaeology and Archaeometry* 18 (2018): pp.97–105.

partly explains the spacing of the zodiac constellations along the ecliptic. For example, the equinoctial constellations for 5600 BCE (Gemini, Sagittarius) are actually shorter than those for 2700 BCE (Taurus, Scorpio).

The OED provides a more comprehensive model to explain this variation. Initially, I used both the modern constellation boundaries and the last-first stars of each constellation as a basis for preliminary RA estimates, from which I was first able to find a correlation between constellation length and the natural division of the OED (see Supplementary Materials Figures SF2 and SF3).[46] However, using today's constellation boundaries (only formalized in 1930) and a modern star map introduces additional uncertainties. In subsequent testing, I therefore compared the OED model with earlier stellar coordinates, initially for 500 BCE and subsequently for 1200 BCE (consistent with MUL.APIN, which gave a better fit), in both cases for 36° N (the estimated latitude where MUL.APIN was composed, also being the latitude favoured by later Greek astronomers).[47] To do this I used estimates of both RA (measured along the equator) and celestial longitude (measured along the ecliptic) based on the last-first stars for each zodiacal constellation using Stellarium software (version 0.12.1). This allowed comparison with the model values based on natural division of the OED.

Using this approach, the OED sectors of each zodiacal constellation were found to separate into three groups of short, medium and long length (Supplementary Table ST4). Of these, the medium constellation sectors of the OED model (Pisces, Aries, Virgo, Libra, by the equinoxes) were not well correlated with the measured RA values when using 2' boundaries B1. Considering only the four short and four long sectors, as identified by the OED model, allows comparison without any assumptions about the setting of boundaries B1 near the equator. Considering only these constellations, I found that those predicted to be long by the OED model were significantly longer than those predicted by OED model to be short. This comparison was significant ($P<0.05$) irrespective of whether the assessment was made using measurements along the equator (RA) or along

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[46] P. Moore, *The Mitchell Beazley Concise Atlas of the Universe* (London: Mitchell Beazley, 1974), inside cover, p.147.

[47] Hunger & Pingree, introduction to MUL.APIN, pp.10–12, based on astronomical grounds. Cf. Hunger & Steele, introduction to MUL.APIN, p.19, who doubt an Assyrian origin of MUL.APIN based on non-astronomical grounds, though this would not exclude that MUL.APIN had been arranged schematically for 36° N.

the ecliptic (longitude). Depending on the assumed form of Sagittarius (whether satyr or centaur; Supplementary Table S5), the comparison gives an estimated 95.4%–98.7% confidence (for RA) and 99.2%–99.9% confidence (for celestial longitude) that the difference is not a chance result.

This finding gave me confidence in the predictive ability of the OED model without any assumptions about the 2' boundaries or the correct form of Sagittarius. In addition, the lack of relationship for the four equinoctial constellations pointed to an incorrect assumption in my first approximation of the line β–γ (2' boundary B1). It indicated the need for an improved rule to join-up the secondary dividing-lines of the six equal constellation pairs (see 2' boundary B2 of Figure 2c). When using boundaries B2, the sector lengths along the NGE are independent of where the model equator is set. In addition, having settled on equator E5 (Supplementary Table ST1), I was also able to test a third rule for the 2' boundaries. In this approach, I continued the 2' boundaries from the poles all the way to the equator, then crossed the equator by 20° RA (2' boundaries B3, Figure 2c). By using either B2 or B3, suddenly all four equinoctial constellations fell in to place. Both these approaches provided a clear separation of the equinoctial constellation sectors in the OED model into those of long duration (Pisces, Virgo: 40° RA) and those of short duration (Aries, Libra: 20° RA).

While all the tested variants were statistically significant (Supplementary Section S2.2e), the closest relationship was found when using the NGE and 2' boundaries B2. In this case, the OED model explains 86% of the variation of zodiacal constellation length as longitude (n=12 pairs, R^2=0.86, P=0.000014, with Sagittarius as a satyr; see Supplementary Materials Figure SF4c). Accepting uncertainty in the form of Sagittarius, if 2' boundary B2 is accepted as justified, then this is equivalent to 99.981–99.999% confidence that the relationship is not a chance result. I also found that the complex symmetry of the OED model is clearly reproduced in the zodiac constellations. The constellations match closely to a characteristic model 'fingerprint' (see Figure 3), which is the result of intersecting the NGE with the partial symmetry of the OED zodiac sectors. Figure 3 shows a 2-way rotational symmetry in the OED model, where opposite zodiac sectors have the same length (Supplementary Table ST4). This is matched by a significant correlation between the actual lengths of opposite constellations, with 96.6–97.1% confidence that this is not a chance result (n=6 pairs; R^2=0.71–0.73; P=0.029–0.034, depending on the form of Sagittarius). This demonstrates ability of the model to predict pattern in the constellation data.

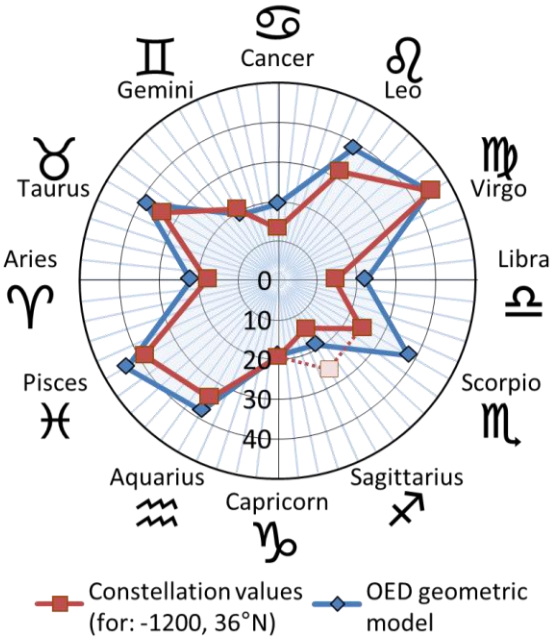

Figure 3: Lengths of the zodiac constellations along the ecliptic (expressed in degrees, °) estimated for 1200 BCE using the difference between first and last stars (brown squares), as compared with values along the natural geometric ecliptic of the orbicular elevated dodecahedron (boundaries B2, blue diamonds). The open square gives the measured value for Sagittarius if this were represented as a 4-legged centaur rather than a 2-legged satyr (Image © Mark Sutton). See Supplementary Figure SF4c for the equivalent x-y plot.

4. A new map of the cosmos

Recognizing the central position of the Path of Anu allowed me to construct a net for the OED as the foundation for a new projection of the celestial sphere (in emulation of the ancients). The key requirement was to adopt equator E5, combined with 5° division of RA and 6° division of δ. Although a critic might suggest that there is arbitrariness in this approach, this is far from the case. As shown in Supplementary Table ST1, equator E5 was selected using four criteria, including consistency with MUL.APIN and basing it on the natural boundaries of the primary scalene triangles. Once the model was plotted flat as a net, it became obvious that equator E5 was the only reasonable option, even though it had not been obvious when considering in three dimensions at the start. Similarly, the choice to map with 5° RA and 6° δ is not chance, but falls out naturally as a *requirement* resulting from the partial symmetry of the OED, when dividing the model into twelve sectors around the poles (see Figure 4).

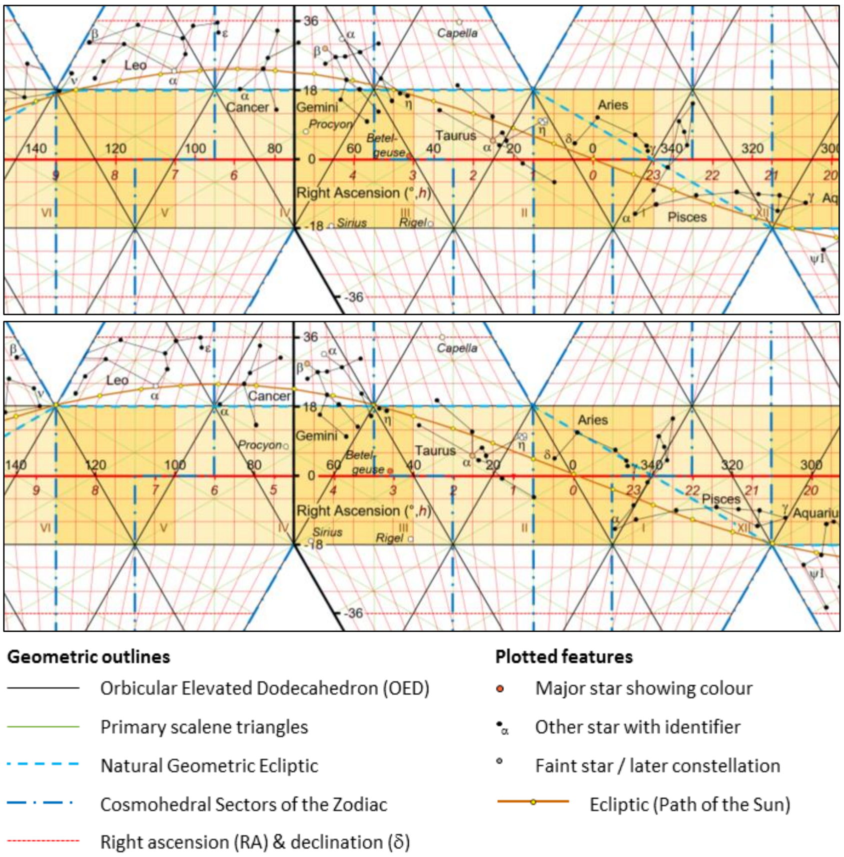

Figure 4: Comparison of two options to set the spring equinox reference (0° Right Ascension) when marking constellations on the orbicular elevated dodecahedron (OED), using data from Stellarium for 1200 BCE. **Top**, 0° RA set as the mid-point of the 30-day sign of Aries (initial approach). **Bottom**, Revised approach, setting 0° RA as the midpoint of the 20-day OED sector of Aries. This improves alignment for Aries with the framework of scalene triangles, as well as several other constellations and the actual ecliptic. For these early maps (plotted October 2013), I showed 2’ sector boundaries B3, and had not yet plotted all the other constellations (Images © Mark Sutton).

In order to represent the constellations on this projection, I set the spring equinox of the ideal year of 360 days (i.e., 0° RA) in mid-Aries in accordance with MUL.APIN and as adopted in the earliest Greek accounts.[48] In mapping the OED for 1200 BCE, I initially set 0° RA in the middle of the 30-day *sign* of Aries. In doing so, I noted that the main line of stars in Aries appeared parallel to, but around 5° in advance of, one of the divisions of the primary scalene triangles, also on the NGE (Figure 4, top). Alignments of Pisces, Taurus, Cancer, Libra and Scorpio were similarly noticeable in being in advance of the OED framework. By revising this to set the 0° RA reference point in the middle of the 20-day OED model *sector* of Aries (i.e., 350° to 10° RA), I found that all these features became aligned to the OED framework, while also shifting the actual ecliptic so that it intersected the vertices of the OED (Figure 4, bottom). Overall, this allowed a close fit between the OED model and the constellations when using Stellarium values for 1200 BC (see Figure 5). Although further work would be warranted in refining approaches to date the OED model, such a tuning to optimize the model fit to the data is fully appropriate, as discussed in Supplementary Section 3.7. Supplementary Figure SF7 shows the effect of intentionally choosing an incorrect date for the model, which substantially reduces its performance in aligning constellations with the OED framework.

The ancient Babylonians also linked the months, Nisannu (~March), Ajjaru (~April), Simanu (~May) etc, with the zodiac constellations (Aries, Taurus, Gemini etc, respectively). If we make this association using the 20° and 40° RA spacing of the OED sectors, then the annual timing of the sun through the Paths of Anu, Enlil and Ea fits exactly to the constellation-months described in MUL.APIN (See Supplementary Results).

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[48] MUL.APIN II.i.19; Hunger & Pingree, notes on MUL.APIN, p.140; Hunger & Steele, notes on MUL.APIN, p.145; E. Dekker, *Illustrating the Phenomena* (Oxford: Oxford University Press, 2013) [hereafter Dekker], pp.32–33.

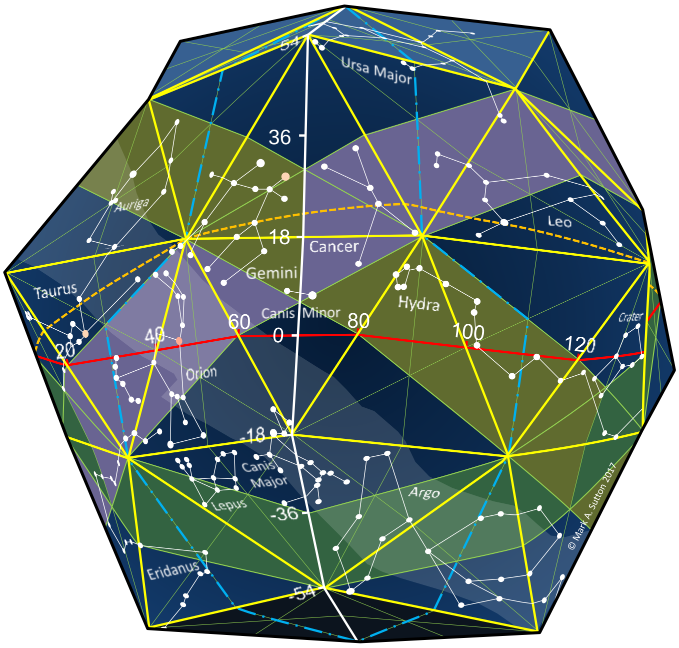

Figure 5: Dodecahedral celestial globe viewed from the outside looking inward, showing the zodiac and other constellations for 1200 BCE. The red line represents the celestial equator (E5), showing values of Right Ascension (RA, °) referenced to the middle of the 20-day sector of Aries. The bold white line indicates declination (δ, °). At the top and bottom, the Arctic Circle (δ 54° N) and Antarctic Circle (54° S) form two hexagons. The central band (δ 18° S to 18° N) matches to the Path of Anu of ancient Mesopotamian astronomy, while the other two bands (δ 18 to 54° N and 18 to 54° S) match to the Path of Enlil and Path of Ea, respectively. The dashed gold line is the ecliptic (Path of the Sun), which matches to key intersections of the celestial grid. The dot-dashed blue lines join the N and S poles illustrating the partial 12-fold symmetry. Three rings (shown as purple, brown and green bands ±10° wide) follow the 10-fold prytany symmetry and appear to have been used to help design the classical constellations (e.g., Cancer, Gemini, Taurus, Hydra, Eridanus etc). (Image © Mark Sutton)

In fact, the degree-of-fit of the zodiac constellations to the OED model varies through the year, with the constellations mapped using Stellarium appearing several days too early during winter (Scorpio to Aries) relative to Sirius. Among several options, this may be partly explained by the OED model of the ancients being based on measurements of VMR, which depends on varying atmospheric visibility and would also cause seasonal mapping differences due to ellipticity of the Earth's orbit around the Sun. Such seasonal shifts are also seen in MUL.APIN and in later Babylonian planetary step-functions (see Supplementary Discussion S3.3).

5. Discussion

5.1. Context and uncertainties

Many uncertainties in this proposed decoration (*diazōgraphōn*) of the dodecahedron may be noted. The experience reminds us of Dionysius, Tyrant of Syracuse, whose own model of the cosmic sphere was supposedly mentioned by Plato in his Second Epistle as not being exactly correct (*Epistle II*, 312D). I can offer no view as to the authenticity of that contested letter, but find it noteworthy that among the reasons that have been given to reject its authenticity is the suggestion that the letter contains 'wanton mystification, of which it is impossible to suppose that Plato could ever have been guilty'.[49] Whether or not the Second Epistle is authentic, such mystification, as well as associated secrecy (314C; cf. *Epistle VII*, 341C), do not appear so surprising in the light of Plato's riddling description at *Timaeus* 55C. The present study of the cosmic dodecahedron paints a picture of Plato as only partially explaining his science, where his accounts might even be seen as reminders for those already familiar with, or initiated into, this kind of learning.[50]

Key uncertainties in my own mapping of the OED include the choice of first and last stars of the constellations, the setting of 0° RA in relation to precession, dating the OED model, and the extent to which the ancients corrected their calculations based on VMR as an approximation of True Morning Rising (which could not be observed because of the rising Sun). An anonymous reviewer has commented: 'A sceptic here would say it takes a considerable amount of arbitrary processing to get the correspondences that the author wants, giving rise to further concerns as to whether these are coincidences or indicate ancient knowledge'. It is a fair criticism and needs to be taken seriously. It is also reminiscent of Kepler's famous polyhedral model of the solar system, nesting the Platonic Solids inside each other. Kepler's polyhedral model actually worked surprisingly well, coming within 3% of the actual spacing for the six known planets

[49] Bury, notes to Plato, pp.312–313, commenting on Plato, *Epistle II*, 312D,E.
[50] cf. Plato, *Phaedrus* 275C; Plutarch, *Life of Alexander* VII.3–5.

Mercury to Saturn[51]. In fact, we can learn a useful lesson by asking why Kepler's model worked so well.

The key seems to be that Kepler had no sound *a priori* theoretical arrangement. He 'tried all possible starting points',[52] considering a near infinity of options using polygons until 'I obtained by chance that which previously I could not reach by any pains'.[53] His solution was to use the Platonic solids. This would have given him yet more opportunity to compare options. With five polyhedra, there are 5 factorial possible orders: 5 x 4 x 3 x 2 x 1, i.e., 120 ways. Putting aside Kepler's later tuning to include spaces between the polyhedra for elliptical orbits, it is perhaps not surprising that he found an option that would work.[54] Of course, Kepler's model could not estimate the spacing of Uranus and Neptune, pointing to limitations of the model's predictive ability.

The example of Kepler's polyhedral model highlights three key elements that we may require in assessing robustness: a) that there should be a sound theoretical/historical foundation which directs and constrains the search, b) that, while the model may legitimately be optimized (Supplementary Figure SF8B), its core performance should be subject to minimum arbitrary judgement, and c) that the model should ideally offer predictive capability for other features yet to be identified. As described in Supplementary Sections 1.3 and 3.7, this list is developed and extended to nine evaluation criteria, providing a comprehensive basis to assess performance of the OED model.

Here it is briefly noted that the core performance of the modelled zodiac contains only one major arbitrary decision: whether to use the 2' boundaries B1 or B2 (Figure 2). As this only affects the results for the near-equinoctial constellations, by excluding these four constellations, no assumptions or arbitrary decisions are needed. In this case, the model is already statistically significant, with 99.2–99.9% confidence (Supplementary Table ST5). By allowing just one arbitrary assumption (to use 2' boundaries B2), the other four constellations all fall into place. Irrespective of decisions about Sagittarius (long, short or excluded), the model spacing along the natural geometric ecliptic explains over 80% of

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51 Field, *Kepler's cosmological theories*.

52 M. Kaspar, *Kepler*, trans. C.D. Hellman (New York: Dover Publications, 1993) [hereafter Kaspar, *Kepler*], p.62.

53 J. Kepler, *Prodromus dissertationum cosmographicarum, continens mysterium cosmographicum de admirabili proportione orbium coelestium* (Tubingae: Georgius Gruppenbachius, 1596), p.8; Kaspar, *Kepler*, p.62.

54 Supplementary Figure SF8A illustrates the statistical problem.

the variation in length of the zodiac constellations, giving at least 99.98% confidence that this is not the result of chance (Figures SF1, SF4, SF5). Supplementary Table ST3 shows that the results are also rather insensitive to which values of RA or longitude to use for the constellations. This is because these lengths are all highly correlated for all the ways of measurement considered, demonstrating the robustness of the approach. To date, no other model has been able to explain the variation in length of the zodiac constellations so well, and with such a coherent theoretical/historical background. It must be for others to show that another model could do better.

5.2. Interpretation and dating of the dodecahedral model

Several messages emerge from this exploration of the OED as an ancient model of the cosmos. Firstly, in the *Phaenomena* of Aratus, based on an account by Plato's associate Eudoxus, it is noted how the six pairs of zodiac constellations are of equal length.[55] This is indeed a feature of the OED model, while we should be careful to note that Aratus did *not* write that all twelve constellations were of equal length, as has often been assumed. Although the arrangement of Aratus' constellations closely match those today, some points of uncertainty remain, such as the extent to which Sagittarius was depicted as a 2-legged satyr (i.e., as a short constellation) more consistent with the OED model, rather than a 4-legged centaur.[56]

Secondly, it can be seen that an alternative arrangement of the OED, locating the celestial poles in the centre of two opposite pentagonal pyramids, would produce a cosmos with a natural 10-month year (see Figure 6). With this orientation, δ naturally follows 5° steps, matching to a description of Martianus Capella,[57] while each 'month' naturally divides into 6, providing the option for 10 months of either 30 or 36 days. It is therefore of interest to note how Plutarch recorded that the Roman year originally had 10 months, making a year of ~300 days.[58] This arrangement also fits with several Mesopotamian records, including: a) an Old Babylonian (2000–1600 BCE) divination ritual with ten stars, b) two

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[55] Aratus, *Phaenomena*, ed. and trans. G.R. Mair (London: Heinemann, 1921) [hereafter Aratus, Mair], 541-544, pp.422–423.
[56] Aratus 306 (Mair, pp.404–405). See the maps at the rear of the Mair edition of Aratus showing Sagittarius as a satyr (cited as deriving from Schaubach's edition of the *Catasterismi* of Eratosthenes).
[57] Martianus Capella, *Marriage* VIII.837 (Stahl et al., p.325)
[58] Plutarch, *Plutarch's Lives*, trans. B. Perrin (London: Heinemann, 1959), *Numa* 18.1–19.6, Vol. 1, pp.367–373.

tablets (Middle and Late Babylonian, 1500–540 BCE) with 10-stars-each for the Paths of Anu, Enlil and Ea, rather than the usual 12-stars-each, and c) a 30-star year indicated by the Greek historian Diodorus Siculus.[59] The other famous 10-month year in antiquity was the civil calendar of 'prytanies' in Athens (c. 500 BCE), with each prytany being approximately 36 days.[60] Given the fondness for using the 'All' of 10 in Athenian civil structures at this time, and this geometrical explanation of the prytanies, we may wonder to what extent such recondite ideas influenced the practices of Athenian democracy.

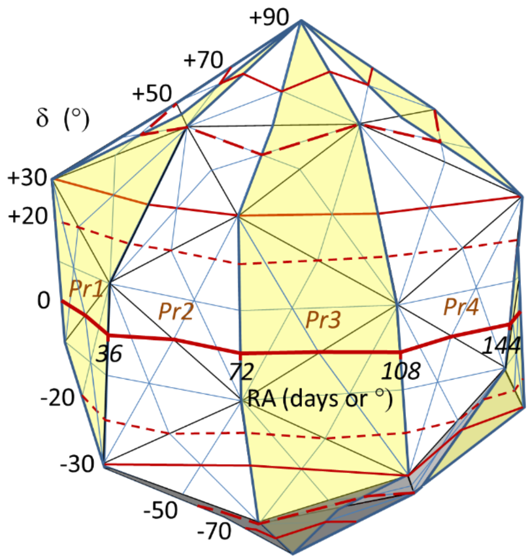

Figure 6: Orientation of the dodecahedral cosmos with celestial poles located in the centres of pentagonal pyramids, dividing the year into ten months or 'prytanies' (Pr1, Pr2 etc). Such an arrangement would place the tropics of Cancer and Capricorn at ±20° and the Arctic/Antarctic circles at ±50° as recorded by Martianus Capella. According to this orientation, with an ideal year of 360 days, RA naturally divides into 6° steps with ten prytanies of 36 days (or 5° steps when using a 300 day year with ten months of 30 days), while δ divides naturally into 5° steps (Image © Mark Sutton).

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[59] Diodorus Siculus, *Library of History* Book 2, 30.6, as noted by J. Oelsner and W. Horowitz, 'The 30-star catalogue HS1897 and the late Parallel BM55502', *Archiv für Orientforschung*. 44/45 (1997/1998): pp.176–185 [hereafter Oelsner & Horowitz].

[60] A.E. Samuel, *Greek and Roman chronology* (München: C.H. Beck, 1972), pp.57–63.

In fact, this 10-way symmetry can be superimposed on the 12-sector OED projection using three different orientations of what might be termed 'Prytany Poles' located at declination ±54°: Cepheus to Carina (i.e., *caput* to keel), Boötes to the ancient river mouth by θ Eridanus, and Ursa Major's front paw to below Sagittarius. This is illustrated in Figure 5 by showing three rings around ±10° of the 'Prytany Equators'. The many alignments with constellation features (e.g., Cancer, Crater, Eridanus, Gemini, Hydra) suggest that the 'Three Rings' were used by ancient astronomers to help them design the classical constellations.

Dating the OED model of the cosmos is especially uncertain given the possibility that its arrangement was modified over time. Based on the *Timaeus* and the constellations of Eudoxus, the model was apparently well-established prior to the 4–5th century BCE. My estimated fitting of the OED model with 0° RA in mid-Aries for 1200 BCE is also consistent with the OED providing a geometric basis for the Paths of Anu, Enlil and Ea. Further evidence for a possible Mesopotamian origin comes from the natural division of the OED meridian into the 60 parts of the sexagesimal system[61] and the setting of the ever-visible Arctic Circle at 54° (Figure 5), implying that 36° N may have been selected as the 'ideal latitude' precisely because of its fit to the OED model. Astronomers may even have travelled to make observations from this ideal latitude deriving from the OED model. Although the present study started from a theoretical reflection on the Platonic Solids, the evidence for Mesopotamian (or other early) interest in the OED is essentially empirical. This means that it is not possible to answer when or how the earliest astronomers became interested in the dodecahedron. It also means that the OED model could easily be much older. In this context, we cannot exclude the possibility that the origins of the 360-day ideal year of the Mesopotamians, their allocation of 360° to the circle[62] and even the sexagesimal system itself, are bound up with the geometry of the OED.

Considering the effect of precession, the earliest date for the OED model arrangement identified here would be ~4000 BCE. Based on overall

[61] Dekker, pp. 32–33; cf. Macrobius, *Dream* II.6.2 (Stahl, p.207); M.A. Powell Jr., 'The origin of the sexagesimal system: The interaction of language and writing', *Visible Language* VI (Winter 1972), pp.5–18; J.V. Torres-Heredia Julca, 'A geometrical link between the circle and sexagesimal system' (2007), https://ui.adsabs.harvard.edu/abs/2007arXiv0707.0676V/abstract [accessed 14 February 2021].

[62] Brack-Bernsen, pp.23–26.

fit of the constellations to the OED, my best estimate for the OED model is around 1200–1100 BCE, perhaps ±400 years, given the uncertainty in setting the reference point for 0° RA (see Supplementary Discussion S3.4). It was only much later that I noted the independent conclusion of Schaefer who used a comprehensive statistical analysis to estimate the astronomical lore in the *Phaenomena* of Eudoxus at 1130±160 BCE (±2σ, i.e., 95% confidence limits), also estimating that it was created for a latitude of 36.0±1.8° N (±2σ).[63] This date is fully consistent with my own fitting of constellations to the OED, and its geometrically 'ideal view' from 36° N. I leave it for others to explore such optimization approaches for dating the OED model according to choice of the 0° RA reference and other criteria linked to the OED alignments.

5.3. Possible origins of the dodecahedral model and ancient cosmological links

A reviewer has asked: what were the features that would have made the OED of interest to the earliest Mesopotamian (or other) astronomers who first used it as a mapping framework? Here it can only be assumed that the ancients, having a wider interest in geometry, had explored multiple polyhedra before selecting the OED.[64] The OED itself would have been particularly attractive from a practical perspective since: a) it is very close to the sphere, b) it has strong symmetry and multiple number relationships, making it very useful as an astronomical model, and c) it is very easy to construct. Indeed, it is the simplest way to produce the outline of a regular dodecahedron, while it also forms a convenient vessel/ship (cf. *holkas*, Philolaus Fragment 12) to hold all the other regular convex polyhedra, although such aesthetic attraction might only have been recognized later.

While the evidence points to Plato as inheriting the dodecahedral universe, his obscurity makes it hard to see what he knew about the mapping framework. Plutarch (*Platonic Questions* 1003D) and Alcinous (*Handbook of Platonism* 13.2) both recognized in the construction of the dodecahedron (from 12 x 30 scalene triangles) an allusion to the zodiac. Newbold went further, suggesting that the 'puzzling words' of *Timaeus* 55C may refer to ancient mapping of the constellations in twelve

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[63] B.E. Schaefer, 'The latitude and epoch for the origin of the astronomical lore of Eudoxus', *Journal for the History of Astronomy* 35 (2004): pp.161–223 (see pp.194–205). For a critical view, see D. Duke, 'Statistical dating of the Phenomena of Eudoxus', *DIO* 15 (2008): pp.7–37 [hereafter Duke].

[64] cf. the great pyramids and other architectural examples. Waterhouse, p.213; Hahn, *Metaphysics*, pp.10–25, 97–101.

pentagonal fields, 'each of which is readily resolved into five triangles'.[65] Yet more curious is the observation that Plutarch associated the Platonic Solids with Plato's five principles of the cosmos: Being/Essence (dodecahedron, All), Identity/Same (octahedron, air), Difference (icosahedron, water), Movement (tetrahedron, fire) and Rest (cube, earth).[66] Of these, Plato (*Timaeus* 34C–36C) had linked the Same and the Different to the celestial equator and ecliptic, respectively. It leaves open the question of whether Plato and his predecessors also linked the dodecahedron with Being/Essence etc, and whether other aspects of Plato's cosmology were intended to correspond to dodecahedral astronomy, or even other disciplines (cf. *Laws* 367D,E).

It is also worth linking the OED with mythological accounts. Drawing on earlier traditions, Porphyry and Macrobius wrote how the Milky Way represents the path of souls, which descend to enter humans in Cancer and ascend back to the heavens in Capricorn.[67] The souls' descent is marked by the brightest star, Sirius (in Canis Major at 18° S), being a part of the Mesopotamian constellation the Arrow [$^{\text{mul}}$KAK.SI.ŠA], which was represented as a downward-pointing arrow belonging to the warrior storm-god Ninurta, who was also identified with Mercury.[68] On the OED, Sirius marks the centre of a giant downward-pointing 'cosmic arrow', formed by combining the entire OED sectors of Gemini and Cancer (see Figures 4, 5).

The star opposite to Sirius in the heavens is Altair, which was already recognized in Mesopotamian astronomy as part of the Eagle constellation [$^{\text{mul}}\text{TI}_8^{\text{mušen}}$], an identity which it continued in Greek astronomy, remaining to this day. The Eagle rises with the Milky Way by Sagitta (the Arrow) and above Sagittarius (Pabilsag),[69] representing the souls' ascent to live as

[65] W.R. Newbold, 'Philolaus', *Archiv für Geschichte der Philosophie* 19 (1906): pp.176–217 (see p.203).
[66] Plutarch, *The Obsolescence of Oracles* 428C–E (Babbit, pp.446–449).
[67] Porphyry, *On the cave of the nymphs*, trans. and introduction, R. Lamberton (Barrytown, NY: Station Hill Press, 1983), 22–23, 28 (pp. 33, 36); Macrobius, *Dream* 12.1–3 (Stahl, pp.133–134).
[68] MUL.APIN I.ii.6, I.ii.16 (Hunger & Pingree, pp.32, 34); J. Black and A. Green, *Gods, demons and symbols of ancient Mesopotamia* (London: British Museum, 1992, reprinted 2004) [hereafter Black & Green], p.35; Oelsner & Horowitz; G. White, *Babylonian Star-Lore* (London: Solaria, 2008) [hereafter White], pp.53–56.
[69] MUL.APIN, I.ii.12, I.ii.33 (Hunger & Pingree, pp.33, 39), cf. I.iii.27–29, as Pabilsag rises, the Arrow (KAK.SI.SA, Ninurta) sets; White, pp.95–97; Aratus 311–315 (Mair, pp. 404–405).

mind or *nous* in the immortal aether. Collectively, these constellations mark the centre (18° N) of a great upward-pointing cosmic arrow combining the OED sectors of Sagittarius and Capricorn. In Babylonian thought, the association appears to be with ascent and the god Marduk (also identified with Jupiter), whose symbol was an upward-pointing arrow (termed ‘spade’ or ‘hoe’).[70] This is also supported by the association of Marduk with the Pleiades (MUL.MUL), Capella (^{MUL}GAM) and Regulus (^{MUL}LU.GAL), given their locations on the OED model.[71] In this context, it should come as no surprise that the Babylonians referred to constellations (MUL, ⁂) as gods, with the single symbol (AN, ✳) meaning heaven, god and the sky god Anu.[72] Further evaluation of the relationships is given in Supplementary Section 3.7 (Criterion 4).

5.4. Towards a coherent interpretation using a paradigm of increasing openness

These multiple associations highlight the close connection between intellectual fields of endeavour in the ancient world, linking each of astronomy, geometry, mythology and philosophy. While such connections are widely recognized, the present examination graphically illustrates Plato’s debt to earlier traditions (whether Mesopotamian, Egyptian or other), with the Pythagoreans as a possible intermediate source.[73] It also recalls accounts that Pythagoras, Plato and Eudoxus all spent significant time in Babylonian-influenced Egypt.[74] It has been suggested elsewhere that Plato was an effective secret keeper.[75] However, given the philosophically sensitive climate of Athens c. 400 BCE and the multi-dimensional nature of these geometrical secrets, it becomes much easier to understand how Plato’s publication of a book like the *Timaeus* could have seen him excommunicated from the Pythagorean community, just as

[70] MUL.APIN I.i.37–38 (Hunger & Pingree, pp.28–29); Black & Green, pp.16, 129.
[71] White, pp.275–276, 305–306, citing star list VR46.
[72] Black & Green, p.30; White, pp.50, 65, 89, 165.
[73] Burkert, *Lore & Science*; Gregory, Notes on Plato’s *Timaeus*; Zhmud, *Pythagoras*; Uždavinys.
[74] DL 8.2–8.3, 8.87, 8.90 (Hicks, pp.321–323, 401, 405); Pliny, *NH* 30.2 (Bostock & Riley, Vol. 5, p.424); Plutarch, *On Isis and Osiris* 354E (Babbit, p.25); cf. Gregory, *Pythagoreans*; Huffman, 2018.
[75] P. Kingsley, *Ancient Philosophy, Mystery and Magic: Empedocles and the Pythagorean Tradition* (Oxford: Clarendon, 1995), p.330.

Neanthes had recorded.[76] The same was recorded of Empedocles by Neanthes and Timaeus of Tauromenium.[77] It further supports the picture of a culture where cosmological secrecy was both important and being challenged.[78]

It is worth relating this interpretation to a view of Burkert that distinguishes a progression in accounts of the mysteries over time from myth to nature allegory to metaphysics, this sequence reflecting a paradigm of 'increasing intellectual elaboration'.[79] While this might broadly reflect historical development in the form of writing, it neglects the significance of a parallel evolution from secrecy toward openness. Based on the constellations and other features of cosmic geography, it appears that the dodecahedral model was known, but, as far as can be seen, was retained unpublished prior to Plato. What was only hinted at by Plato later becomes more explicit in Plutarch's description of the 360 primary scalene triangles, even if Plutarch himself still playfully retained some features without full explanation.[80]

My conclusion is that such accounts also need to be interpreted in relation to a concurrent 'paradigm of increasing openness', with the implication that 'discoveries' may be much older than the earliest published accounts. In this context, oft-used 'arguments from silence' become worse than unreliable.[81] Distinguishing new discovery from

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[76] Janko, *Derveni papyrus.*

[77] DL 8, 54–55 (Hicks, p.371).

[78] For example, Plutarch, *The Obsolescence of Oracles* 417B,C; Clement, *Stromateis* 5.7–9; Horky, *P&P*, pp.116–122; S. Schorn, 'Pythagoras and the historical tradition: from Herodotus to Diodorus Siculus', in C. Huffman, ed., *A History of Pythagoreanism* (Cambridge: Cambridge University Press, 2014), pp.296–314 [hereafter Schorn] (pp.303–310).

[79] W. Burkert, *Ancient mystery cults* (Cambridge, MA: Harvard University Press, 1987), pp.72–73; cf. Zhmud, *Pythagoras*, p.17; Gregory, *Pythagoreans*, p.31.

[80] Plutarch, *Platonic Questions* 1003D; *The Obsolescence of Oracles* 422–428; Opsomer, pp.42–43.

[81] This could help resolve the difficulty of Duke, pp.15–16, concerning the early dating of Eudoxus' *Phaenomena*, also recalling the tradition that Eudoxus studied in Egypt and under the Pythagorean, Archytas. On the dynamics and symbolic capital of secrecy, cf. H.B. Urban, 'The torment of secrecy: Ethical and epistemological problems in the study of esoteric traditions', *History of Religions* 37 (1998): pp.209–248; A. Lenzi, 'Advertising secrecy, Creating power in ancient Mesopotamia: How scholars used secrecy in scribal education to bolster and perpetuate their social prestige and power', *Antiguo Oriente* 11 (2013): pp.13–42 [hereafter Lenzi]; Struck, pp.160–161.

increasing openness as part of a gradual democratization of knowledge is necessarily far from easy.[82] However, this perspective may provide a useful lens for future research, recognizing that time and changing geopolitical context would have altered the limits of acceptable publication.

Expecting that some may find the present conclusions contentious, the nine evaluation criteria provide a useful way to assess the results. Seen altogether, they present a robust case that the OED was used as a mapping framework (Supplementary Discussion S3.7). This extends a parallel conclusion based on written sources from Plutarch to Ficino, that the OED was still recognized as a cosmological model.[83] The further conclusion, that the dodecahedral model implies ancient knowledge of geometry, is based mainly on inference, since the OED is self-evidently a geometric construct. By contrast, the present study offers no evidence for early knowledge of advanced geometry, such as Euclid's construction of the regular dodecaedron (*Elements* 13.17), nor of any geometrical proofs. Assembling the OED requires only the rudiments of geometry (using equilateral triangles), with measurement across its surface using simple arithmetical approaches. Indeed, this simplicity may have been part of its attractiveness. The OED is thus the easiest way to construct the 'regular dodecahedron that is a part of its composition, as the intellect by imagination alone understands'.[84] Such reflections point to many opportunities for deeper investigation into the fullness of Plato's dodecahedral cosmology.

Of recent commentators, Horky comes closest to what we see here.[85] Arguing for a dependence on Aristotle, he takes seriously Iamblichus' oft-doubted record that the Pythagorean Hippasus was punished for revealing the secret of how to construct the 'sphere of twelve pentagons'.[86] While Horky does not allude to the central cosmological role of the dodecahedron, his wider narrative makes clear that this record should be seen as part of an ongoing democratization of knowledge.

If the importance of the OED has remained 'reserved knowledge' until now, it is equally fair to say that Plato's hints are not immediately obvious to the uninitiated. Similarly, given their cryptic way of writing, it is hard

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[82] On democratization, see Horky, *P&P*, pp.112–126; Lenzi, p.34; Schorn, p.304.
[83] *Elevated dodecahedron*, forthcoming.
[84] Pacioli, *Divina Proportione*, p.15v. (trans. Matt Pearce); cf. *Phaedrus* 247C.
[85] Horky, *P&P*, pp.57–60, 84–88, 112–126.
[86] Iamblichus, *Pythagorean Life* 18 (Taylor, p. 232); cf. Burkert, *Lore & Science*, pp.457–465; Netz, *Problem*, p.181.

to say exactly how much Plato, Plutarch or the later Neoplatonists knew of the OED's cosmology. Under these circumstances, we should not be surprised that this system could remain hidden for several millennia. While there were occasional glimpses, such as two elevated dodecahedra in the floor of St Mark's basilica in Venice (attrib. to Paolo Uccello, c. 1397–1475, but possibly later),[87] those in the know were certainly not explaining themselves too openly.[88] These observations all point to the likely existence of other forms of ancient science that have yet to be lifted from obscurity.

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Data Accessibility
The datasets supporting this article are included in Supplementary Material, accompanying this paper at www.CultureAndComsos.org

A complete set of maps is available at https://www.euipo.europa.eu/, Registered Community Designs 008837850-0001 to -0009, and at https://www.registered-design.service.gov.uk/ registered designs 6187789 to 6187797.

[87] J.V. Field, 'Rediscovering the Archimedean Polyhedra: Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Dürer, Daniele Barbaro, and Johannes Kepler', *Archive for history of exact sciences* 50 (1997): pp.241–290; E. Vio, ed., *Il manto di pietra della basilica di San Marco a Venezia. Storia, restauri, geometrie del pavimento* (Venezia: Cicero, 2012), pp.59, 152 and CD-ROM.
[88] Cf. R. Paier, *Simbole e Misteri nelle geometri del pavimento di San Marco a Venezia* (Venezia: Grafiche Veneziane, 2011), p.280.