Harmony vs Symmetry:
Kepler's view and the role of *Spielraum*

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Abstract: In a famous passage in *De revolutionibus*, Copernicus remarked that 'in this arrangement [*ordinatione*] ... we discover a marvellous symmetry of the universe [*mundi symmetriam*], and an established harmonious linkage [*harmoniae nexum*] between the motion of the orbs and their size, such as can be found in no other way'.[1] Copernicus has brought together two previously distinct aesthetic values: symmetry as proportionality in what is efficient or pleasing to the eye; and harmony as proportionality in what is pleasing to the ear. This is a critical passage where two aesthetic criteria are put to use to capture two different aspects of the universe: its design and its motion. Symmetry captures the design, that is, the relation of the parts (the planetary orbs) to the whole (the Universe), whereas motion (understood as the planetary periods) is linked to size (understood as the planetary distances from the Sun). What was Kepler's view of these two distinct aesthetic criteria? I conclude that Kepler did not invoke the criterion of symmetry in any of his writings and appealed only to harmony, but he had a sophisticated view of this concept which required—so my argument goes—a certain degree of freedom which I call *Spielraum*. This view is in stark opposition to that of Galileo's.

Harmony versus symmetry: Kepler's view and the role for *Spielraum*
In the dedication to *De revolutionibus*, Copernicus rebukes traditional astronomers for failing to follow sound principles:

> Their experience was just like someone taking from various places hands, feet, a head, and other pieces, very well depicted, it may be, but not for the representation of a single person; since these fragments would not belong to one another

[1] E. Rosen, *Nicholas Copernicus On the Revolutions* (Baltimore, MD, and London: John Hopkins University Press, 1992), p.22.

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> at all, a monster rather than a man would be put together from them.[2]

When disparate elements are put together the result is monstrosity, rather than a beautiful human form. Copernicus thinks of Ptolemy's models as a mixed bag of theories which together depict the universe as 'a monster'. Moreover, these astronomers could not

> deduce… the principal consideration, that is, the design of the universe [*mundi formam*] and the true symmetry [*symmetriam*] of its parts.[3]

In other words, Copernicus suggests that, although the heavens have a proper architectural design, traditional astronomers have failed to grasp it. He reports that

> For a long time, then, I reflected on this confusion in the astronomical tradition concerning the derivation of the motions of the universe's spheres. I began to be annoyed that the movements of the world machine [*machinae mundi*], created for our sake by the best and most orderly Artisan of all, were not understood by the philosophers...[4]

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[2] Rosen, *Nicholas Copernicus On the Revolutions,* p.4; Nicolaus Copernicus, *De revolutionibus*, (Nuremberg, 1543), f. iij v: 'Sed accidit eis perinde, ac si quis è diuersis locis, manus, pedes, caput, aliaque membra, optime quidem, sed non unius corporis comparatione, depicta sumeret, nullatenus inuicem sibi respondentibus, ut monstrum potius quàm homo ex illis componeretur.' In his commentary, Rosen (p.341) suggests that Copernicus took this image from the first five lines of Horace's *Art of Poetry* in which he describes a 'monster' formed from the parts of different kinds of animals, but there is nothing in that passage about proportions or symmetry. Moreover, Copernicus speaks of a human form, not of an animal.
[3] Rosen, *Nicholas Copernicus On the Revolutions.* p.4 (slightly modified). Copernicus, 1543, f. iij v: 'Rem quoque praecipuam, hoc est mundi formam, ac partium eius certam symmetriam non potuerunt inuenire...'
[4] Rosen, *Nicholas Copernicus On the Revolutions,* p. 4 (slightly modified). Copernicus, 1543, f. iij v: 'Hanc igitur incertitudinem Mathematicarum traditionum, de colligendis motibus sphaerarum orbis, cum diu mecum reuoluerem coepit me taedere, quòd nulla certior ratio motuum machinae mundi, qui propter nos, ab optimo & regulariss[imo], omnium opifice, conditus esset, philosophis constaret...'

Despite the lack of an explicit reference to Vitruvius, Bernard R. Goldstein and I suggest that Copernicus expected the universe to comply with the Vitruvian concept of symmetry: a temple (the universe) whose constituent elements (the planetary orbs) relate to each other to form a beautiful whole.[5] Indeed, Vitruvius uses the term *symmetry* to refer to the well-proportioned features of the human body, the structure of a building, and the efficient functioning of a machine. In *De architectura*, he treats separately these three domains in which symmetry is applied.[6]

Copernicus alludes to these three aspects of symmetry, but he considers them together. He has thus applied symmetry in a new way while still retaining Vitruvius's 'principle of symmetry'.[7]

At the juncture where Copernicus claims to have grasped this 'principal consideration', he asserted that

> In this arrangement [*ordinatione*]… we discover a marvelous symmetry of the universe [*mundi symmetriam*], and an established harmonious linkage [*harmoniae nexum*] between the motion of the orbs and their size, such as can be found in no other way.[8]

Copernicus brings together two previously distinct aesthetic values: *symmetry* as proportionality in what is efficient or pleasing to the eye; and *harmony* as proportionality in what is pleasing to the ear. The notion of proportionality in sound which pleases the ear is well attested in classical writings. In some ancient texts motion was associated with sound and hence with harmony.[9] Copernicus associates motion with harmony but, in contrast to his likely sources, says nothing about sound. The omission of

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[5] Giora Hon and Bernard R. Goldstein, 'Symmetry in Copernicus and Galileo', *Journal for the History of Astronomy* 35 (2004): pp.273–292.

[6] Giora Hon and Bernard R. Goldstein, *From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept* (Dordrecht: Springer, 2008), pp.100–106.

[7] Vitruvius, *De architectura*, IX.1, 2; see F. Granger, (ed. and trans.), *Vitruvius: On Architecture* 2 vols (Cambridge, MA: William Heinemann, 1962), pp.ii, 212–13. For an extensive discussion of the concept of symmetry in ancient and early modern times, see Hon and Goldstein, *From Summetria to Symmetry*.

[8] Rosen, *Nicholas Copernicus On the Revolutions*, p.22 (slightly modified). Copernicus, 1543, I.10, f. 10a: 'Inuenimus igitur sub hac ordinatione admirandam mundi symmetriam, ac certum harmoniae nexus motus & magnitudinis orbium: qualis alio modo reperiri non potest.'

[9] See Hon and Goldstein, *From Summetria to Symmetry*, pp.159–160.

sound may have led some of his readers to take *harmony* as a synonym for *symmetry* since both involved proportion.

This is a critical passage where two aesthetic criteria are put to use to capture two different aspects of the universe: its design and its motion. Symmetry captures the design, namely, the relation of the parts (the planetary orbs) to the whole (the universe), whereas motion (understood as the planetary periods) is linked to size (understood as the planetary distances from the Sun). The key to 'the established harmonious linkage' is that the periods of the planets are longer as their orbs are farther from the centre of motion. This principle was stated by Vitruvius in a geocentric context:

> the farther distance … [the planets] are from the limits of heaven and the nearer they keep their path to Earth, the faster they seem to go, because each one of them, in traversing a smaller circle, more frequently passes underneath one which is higher up, and then overtakes it.[10]

By adopting this principle, namely, that planets farther from the Sun (now taken as the centre of motion) move more slowly, Copernicus has linked harmony with symmetry on a cosmic scale as a feature of a perfect structure. *Symmetry* here is based on the claim that, according to Copernicus, the parts of the cosmos fit together to form a perfect whole, on analogy with what Vitruvius discerned in the human form and demanded of temples and machines.

Since Goldstein and I have not found *harmony* associated with *symmetry* in any source available to Copernicus, we suggest that this combination of *harmony* and *symmetry* is due to him.[11] Copernicus takes advantage of the aesthetic value of both *harmony* and *symmetry*; thus, for

[10] Vitruvius, *De architectura*, IX.1, 14; see I. D. Rowland, T. N. Howe, and M. J. Dewar (transl.). *Vitruvius: Ten Books on Architecture* (New York: Cambridge University Press,1999), p.111. Aristotle had presented a similar argument in *De caelo*, ii.10 (see W. K. C. Guthrie, *Aristotle: On the heavens* (Cambridge, MA: Loeb Classical Library, [1939] 1960), p. 199) which was reinterpreted by Averroes and his school. However, while Aristotle talks about the planetary distances from the prime mover, Vitruvius considers the distances with respect to the Earth which, for him, is the centre of motion. For additional details, see B. R. Goldstein, 'Copernicus and the Origin of his Heliocentric System', *Journal for the History of Astronomy* 33: (2002): pp.219–235; p.225.

[11] See Hon and Goldstein, 'Symmetry in Copernicus and Galileo'.

him, *symmetry* does not have the Euclidean sense of commensurability.[12] In fact, he does not call attention to the planetary distances that could have provided the measure of cosmic commensurability. Be that as it may, Copernicus maintains the distinction between these two terms although his readers began to treat them as if they were synonyms.

A standard claim in modern astrophysics is that the very large can be seen in the very small.[13] Kepler already considered this idea in 1611 in *A New Year's Gift or On the Six-Cornered Snowflake* (*Strena Seu De Niue Sexangula*), a gift to his benefactor, John Matthew Wacker (1550–1619). 'I am exhibiting,' Kepler writes, 'the soul of ... the globe of the Earth, in the mote of a snowflake!' From the tiny, ephemeral snowflake, 'from this almost Nothing I have almost formed the all-embracing Universe itself!'[14] Poetic license aside,[15] the *Gift* is a path-breaking study in the mathematics of morphogenesis. Kepler observes the hexagonal form of the six-cornered snowflake and asked:

> Why, whenever snow begins to fall, its initial formations invariably display the shape of a six-cornered starlet? For if it happens by chance, why do they not fall just as well with five corners or with seven? Why always with six, so long as they

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[12] The standard source for mathematical usages in ancient Greek is, of course, Euclid's *Elements*, and symmetry is indeed a technical term in this treatise. Symmetry, as its etymology indicates, can mean commensurability—'in measure with', or 'sharing a common measure'—as in the definition that Euclid provides. For details, see Hon and Goldstein, *From Summetria to Symmetry*, pp.70–71.

[13] For a brief account of this idea in modern cosmology, see John North, *Astronomy and Cosmology* (New York and London: Norton, 1995), p.597ff.

[14] Colin Hardie, ed. and trans., *The Six-Cornered Snowflake*, *Latin text edited and translated by Colin Hardie, with essays by Lancelot L. Whyte and Basil John Mason* (Oxford: Clarendon Press, 1966), p.39; Johannes Kepler, *Strena Seu De Niue Sexangula* (Frankfurt am Main: Tampach, 1611), p.21; Johannes Kepler, M. Caspar et al., eds, *Johannes Keplers gesammelte Werke,* Vol. 4 (München: C. H. Beck, 1937–), p.277: 'quia ex hoc pene Nihilo pene Mundum ipsum, in quo omnia, efformaui: ... iam ter maximi Animalis, globi telluris, animam in Niuis Atomo exhibeo?' Kepler may be playing with the Renaissance theme of man as a microcosm of the universe (i.e., the macrocosm). The literature on this subject is vast: see, e.g., George Boas, 'Microcosm and macrocosm.' in Philip P. Wiener, ed., *The dictionary of the history of ideas,* 4 vols. (New York: Scribner's, 1973), 3: pp.126–131.

[15] As Cecil J. Schneer, 'Review of Hardie (ed. and tr.) 1966.' *Isis* 58 (1967): pp.134–136 remarks, the style of Kepler's *Gift* should not be ignored.

are not tumbled and tangled in masses by irregular drifting,
but still remain widespread and scattered?[16]

In Kepler's view there must be some definite cause for this phenomenon. He therefore proposes 'to inquire into the origin of this shape in snowflakes and to decide between external and internal causes.'[17] To pursue his inquiry Kepler recasts the problem in terms of packing or space filling, and drew analogies from the shapes of honeycombs and pomegranates.[18] He examines several tentative solutions but expresses dissatisfaction with them. Finally, he passes the problem on to chemists (*dicant igitur Chymici*), suggesting that the solution may depend on the idea of *facultas formatrix*, that is, an inherent formative faculty in matter itself.[19] He asks, 'Does the nature of this formative faculty partake of six-corneredness in the inmost recess of its being?'[20] With this clear formulation of the problem, and an outline of a possible solution, Kepler indicates the need for a science of the formation of visible forms in crystals, plants, and animals.

In this original study Kepler nowhere invokes the term, *symmetry*; he did not see it necessary to appeal to *symmetry* for presenting the problem or trying to solve it. Kepler was not shy about coining new terms; had he seen the need for a new term, he would have invented it, or adapted an old

[16] Hardie, *The Six-Cornered Snowflake*, p.7; Kepler, 1611, 5; Caspar et al., eds, *Johannes Keplers gesammelte Werke,* 4: p.265: 'Cum perpetuum hoc sit, quoties ningere incipit, vt prima illa Niuis elementa figuram prae se ferant Asterisci sexanguli, causam certam esse necesse est. Nam si casu fit, cur non aequè quinquangula cadunt, aut septangula, cur semper sexangula, siquidem nondum confusa et glomerata multitudine, varioque impulsu, sed sparsa & distincta?'

[17] Hardie, *The Six-Cornered Snowflake*, p.21; Kepler, 1611, p.12; Caspar *et al.*(eds.), *Johannes Keplers gesammelte Werke,* 4: p.271: 'Cum enim proposuissemus inquirere originem figurae huius in niue inter causas extrinsecas et intrinsecas:...'

[18] Hardie, *The Six-Cornered Snowflake*, pp.9–13; Kepler 1611, pp. 6–8; Caspar *et al.*(eds.), *Johannes Keplers gesammelte Werke,* 4: 265–267.

[19] Hardie, *The Six-Cornered Snowflake*, pp. 41–45; Kepler 1611, pp.22–24; Caspar et al., eds, *Johannes Keplers gesammelte Werke,* 4: pp.278–280. On the expression, *facultas formatrix*, see Lancelot L. Whyte, 'Kepler's unsolved problem and the *facultas formatrix*' in Hardie, *The Six-Cornered Snowflake.* 'Chemist' in Kepler's time was not distinguished from 'alchemist'.

[20] Hardie, *The Six-Cornered Snowflake*, p.41; Kepler 1611, p.22; Caspar *et al.*(eds.), *Johannes Keplers gesammelte Werke,* 4: 278: 'An denique ipsa huius formatricis Natura in intimo sinu suae essentiae particeps est sexanguli?

one (see, for example, *orbit* and *focus*).[21] Still, modern commentators persist in appealing to the modern scientific concept of symmetry in their comments on this text. For example, Basil John Mason, an authority on weather phenomena, remarks that Kepler recognizes the hexagonal symmetry of snow crystals. And 'although Kepler was unable to offer a satisfactory explanation of the six-sidedness of the snowflake, his discussion of space-filling and symmetry laid the early foundations of crystallography.'[22] Similarly, Cecil J. Schneer, the reviewer of the translation of Kepler's *Six-Cornered Snowflake* from Latin to English, indicates that the Latin, *sexangula*, is

> closer to the modern crystallographer's 'hexagonal' with its implication of symmetry—although the concept of symmetry is an outgrowth of just this kind of speculative essay rather than a contribution to it.[23]

Indeed, Kepler's account can be recast in terms of the modern concept of symmetry, but this is history in reverse.

Kepler does not appeal to symmetry and takes the route of harmony. His views on cosmic harmony were elaborated in many of his works, especially *Harmonices mundi* in 1619 whose publication we celebrated in 2019. In this work Kepler presents his third law; it describes a strict relation of the planetary periods with their distances.

> R, the mean distance from the Sun, is the independent variable, and T, the period, is dependent on it: $T \propto R^{3/2}$.[24]

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[21] Bernard R. Goldstein and Giora Hon, 'Kepler's move from orbs to orbits: documenting a revolutionary scientific concept,' *Perspectives on Science* 13 (2005): p.92, n. 20.

[22] Basil John Mason, 'On the Shapes of Snow Crystals; a commentary on Kepler's essay 'On the Six-Cornered Snowflake' in Hardie, *The Six-Cornered Snowflake,* p.52. Cf. John G. Burke, *Origins of the science of crystals* (Berkeley and Los Angeles, CA: University of California Press, 1966), p.35; Cecil J. Schneer, 'Kepler's New Year's Gift of a Snowflake', *Isis* 51 (1960): p.543.

[23] Schneer, 'Review of Hardie', p.134.

[24] Bernard R. Goldstein, 'What's New in Kepler's New Astronomy?' in John Earman and John D. Norton, eds, *The Cosmos of Science: Essays of Exploration* (Pittsburgh: University of Pittsburgh Press, 1997), p.18.

The discovery of this law crowned a lifelong search for harmonic relation on a cosmic scale.

As indicated, Goldstein and I claim that the link between symmetry and harmony, introduced by Copernicus, was innovative.[25] Kepler, however, does not follow Copernicus in this regard and does not link harmony with symmetry. In fact, Kepler does not appeal to symmetry at all. He ignores Copernicus's invocation of *symmetry* and elaborates the sense of cosmic *harmony*. Indeed, his views on cosmic harmony had already appeared in his first major work, *Mysterium Cosmographicum* (1596), and were developed in several of his later works. But his precise sense of harmony is only loosely connected with the critical passage in Copernicus. For example, in 1620 two years after the discovery of his third law, Kepler explains in his *Epitome of Copernican Astronomy*:

> The archetype of the movable world is constituted not only of the five regular [solid] figures—by which the paths of the planets and the number of the courses were determined—but also of the harmonic proportions with which the courses themselves were attuned, as it were, to the idea of celestial music or of a harmonic concord of six voices. Now since this musical ornamentation demanded a difference of movement in any given planet—a difference between the slowest and the fastest movement; and this difference is made by the variation of the interval between the planet and the sun; and since the magnitude or ratio of this variation was required to be different in different planets; hence *it was necessary that some very small amount should be taken away* from the intervals which are exhibited by the figures as uniform and without variation, and that *it should be left to the freedom of the composer to represent the harmonies of movement... nevertheless that which the regular solids have of their very own was not neglected in this very small discrepancy*.[26]

[25] Hon and Goldstein, 'Symmetry in Copernicus and Galileo'.

[26] Johannes Kepler, *Epitome Astronomiae Copernicanae* [*Epitome of Copernican Astronomy*] (Linz: Tampachius, 1618–1621), p.871 [italics added], Reprinted in M. Caspar et al., *Johannes Keplers gesammelte Werke*, Vol. 7. See Charles G. Wallis, trans., *Johannes Kepler: Epitome of Copernican Astronomy, Books IV and V* in Robert M. Hutchins, ed., *Great Books of the Western World*. Vol. 16: *Ptolemy, Copernicus, Kepler* (Chicago: Encyclopaedia Britannica, 1952): 16: 839–1004.

To take 'some very small amount' away from the intervals and apportion it, as it were, to 'the composer' so that he could freely tune the movements to make them harmonious, necessitates a certain degree of freedom. Put differently, in Kepler's world picture nature requires some leeway, some latitude; such a world has an inherent *Spielraum*, to use a German term. Crucially, then, the Platonic figures do not determine on their own the intervals between successive planets, for the intervals are also dependent on the ornament of harmonic movements, which requires a certain amount of freedom.

In his concluding words of the *Harmonice mundi*, Kepler reflects on the development of his astronomical research. Having introduced the five Platonic solids, he could account for both the number of the planets and nearly the right size for the intervals among them; for the remaining discrepancy he appeals to the state of accuracy of astronomy. In the course of twenty years, the accuracy of astronomy, Kepler remarks, had been perfected and yet there was still a discrepancy among the distances and the solid figures—the reasons for the very unequal distribution of the eccentricities among the planets being not yet apparent. Kepler writes:

> Just as the bodies of animate beings have not been made, and a mass of stone is not usually made, according to the pure norm of some geometrical figure, but *something is removed from the external round shape*, however elegant (though the correct amount of bulk remains) so that the body can take on the organs necessary to life, and the stone the likeness of an animate being, similarly also the proportion which the solid figures were to prescribe for the planetary spheres, as lower, and having regard only to a body of a particular size and to matter, must have given way to the harmonies, as much as was necessary for the former to be able to stand close and to adorn the motions of the globes.[27]

Kepler argues that the explanation for the unequal distribution of the eccentricities among the planets has been obtained by removing something from the pure form. This implies that nature has some degree of freedom, some fuzziness, which no mathematics could determine. The scientist, as

[27] Johannes Kepler, 1997. *The Harmony of the World* (Harmonice mundi, 1619), E. J. Aiton, A. M. Duncan and J. V. Field, trans, with an introduction and notes (Philadelphia, PA: the American Philosophical Society, 1997), 209:489-490 [italics added].

a calculator, has to realize a *Spielraum* in the system and acknowledge its effects. It is this leeway that allows for the nesting hypothesis and for the harmony of the planetary motions.[28] Kepler seems then to regard nature as having an inherent gap. To be positioned in the solar system, a planet must be prescribed a solid figure, but this is not enough, for to move harmoniously it would require some degree of freedom—a *Spielraum*.[29]

We need not dwell here on the musical aspect of Kepler's conception of harmonic concords.[30] Suffice it to say that Kepler maintains his enthusiasm for harmonic relations without linking the concepts of harmony and symmetry, as Copernicus had done. John Keill (1671–1721), Professor of Astronomy at Oxford in the early 18th century, attests to the fact that Kepler divorced his physics from the Copernican linkage of harmony and symmetry. This evidence has the advantage that it comes from a source uncontaminated by recent treatments of the subject. In his astronomical lectures, Keill remarks that

> Comparing the Periods of the Planets, or the Times they take to finish their Circulations, with their Distances from the Sun, we find they observe a wonderful harmony and proportion to one another.[31]

Keill rightly refers to Kepler's third law as 'harmony'. *Symmetry* does not have an entry in the Index of Keill's published lectures either in the Latin or the English versions.

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[28] For the nesting hypothesis, see Goldstein and Hon, 2018.

[29] For an extensive discussion of this concept in Kepler's conception of nature, see Hon, 2004.

[30] Kepler invokes musical concords to justify modifications of the planetary distances implied by his hypothesis of nested regular solids so that these distances would agree with the data derived from observations. See, e.g., Albert Van Helden, *Measuring the Universe: Cosmic Dimensions from Aristarchus to Halley* (Chicago, IL, and London: University of Chicago Press, 1985), chs. 6 and 8.

[31] John Keill, *An introduction to the true astronomy, or, Astronomical lectures read in the astronomical school of the University of Oxford* (London: Lintot, 1721), p.23; John Keill, *Introductio ad veram astronomiam, seu, Lectiones astronomicæ: habitæ in schola astronomica Academiæ Oxoniensis* (Oxford: Clements, 1718), p.36: 'Comparatione factâ, miram quandam inter Planetarum Tempora, quibus circuitus suos circa Solem absolvunt, & ipsorum à Sole distantias deprehendimus harmoniam, & Proportionem.'

As is well known, Kepler's youthful *Mysterium cosmographicum* is the key text, for Kepler claims that the seeds of his later discoveries were already in evidence there. In the original edition of the *Mysterium* Kepler concludes with a rejection of a common period for all the planets, in contrast to the tradition of the Platonic Year that first appeared in Plato's *Timaeus*. The Platonic Year, or World Year, is the common period for the return of all the planets to their original configuration—the default of the world as it had been originally designed by the creator.[32] According to Kepler, for astronomy to be perfect, it 'ought to adopt hypotheses which would be satisfactory if the universe were eternal.'[33] On the basis of a single assumption, Kepler claims to have proved that there is no Platonic Year. The argument was based on the claim that 'the motions are in irrational [*irrationales*] proportions to each other, and thus they will never return to the same starting point, even if they were to last for infinite ages.'[34]

In 1621 Kepler returns to this issue in the second edition of the *Mysterium* and asks, 'Is some exact return of all the motions to their starting point to be found?'[35] Kepler raises the question since he realized that with his third law of planetary motion the argument in the original edition of the *Mysterium* had been refuted. For, according to this law, if T_1 and T_2 are the periods of two planets and R_1 and R_2 are their mean distances from the Sun, then the ratio of T_1 to T_2 is equal to the ratio of the $^3/_2$ powers of R_1 and R_2.[36] The question therefore persists, is there an argument, consistent with the third law, that undermines the Platonic Year? Kepler's response to his rhetorical question is worth citing in full (ch. 23):

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[32] Plato, *Timaeus*, trans. R.G. Bury (Cambridge, MA, and London: Harvard University Press, 1931), 39D.
[33] Alistair M. Duncan, ed. and trams. *Johannes Kepler: Mysterium Cosmographicum (The Secret of the Universe)* (New York: Abaris, 1981), p.183 (slightly modified); Kepler [1596] 1621, p.67.
[34] Duncan, *Johannes Kepler,* p.223; Kepler [1596] 1621, 87: ch. 23 in the original edition.
[35] Duncan, *Johannes Kepler*, p.225; Kepler, [1596] 1621, 88, author's notes in edn. 1621 to ch. 23 in edn 1596.
[36] For Kepler's reasoning leading to his third law see, e.g., Bernard R. Goldstein, 'What's New in Kepler's New Astronomy?' In John Earman and John D. Norton, eds, *The Cosmos of Science: Essays of Exploration* (Pittsburgh, PA: University of Pittsburgh Press, 1997), pp.18–20.

> The mean motions are formed from the arithmetic mean between the extremes, [i.e., the motions at] aphelion and perihelion, and that mean between these expressible [i.e., rational] terms is expressible [*effabiles*]. [On the other hand,] they are also formed from the geometric mean between the same terms. But the geometric mean between expressible terms is not always expressible. Therefore the mean motions of the planets are inexpressible [*ineffabiles*], and incommensurable [*incomemensura-biles*] with the extreme motions [in the case] of all the planets... However, since *a priori* there is no proportion which controls the mean motions, but they spring individually from their own extreme motions, the mean motions will not be commensurable [*medij motus ne inter se quidem commensurabiles*] even among themselves; for no regular property, such as expressibility, normally exists by accident [*casu*]. Therefore no exact return of the motions to their starting point is to be found... [37]

Note that earlier in the same passage, Kepler explicitly states that he prefers 'expressible' and 'inexpressible' over the usual mathematical terms 'rational' and 'irrational'.[38] Kepler's argument is new—no one before him was interested in the ratio of mean to extreme motions. However, Kepler's claim depends on an assessment of a probability and, in that sense, it seems not unlike the medieval discussions. Notwithstanding the differences in approach, the same issue is addressed in the tradition in which incommensurability was considered consistent with nature.[39] In other words, nature exhibits ratios that cannot be expressed as a ratio of integers.

Galileo, by contrast, does not seem to respond to this tradition. Unlike Kepler, he occasionally invokes the term *symmetry*, but not in the mathematical sense it has in the Greek text of Euclid's *Elements*. Furthermore, Galileo finds the possibility of incommensurability in nature to be problematic. In a letter dated 16 July 1611, Galileo seeks to resolve

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[37] Duncan, *Johannes Kepler*, p.225 (slightly modified); Kepler [1596] 1621, 88, author's notes in edn. 1621 to ch. 23 in edn 1596.

[38] '*Illarum enim quatuor proportiones sunt ineffabiles, seu vt hic cum vulgo appellant irrationales*' (Kepler [1596] 1621, 88). As Thomas L. Heath, trans., *The Thirteen Books of Euclid's Elements*, 3 vols (New York: Dover, [1926] 1956), 3: 12 notes, Euclid's term for *rational* (*rhêtos*) literally means 'expressible'.

[39] See, e.g., the case of Oresme (Hon and Goldstein, 2008, pp.79–82).

the tension between finite reason and unlimited, concrete, physical reality, and thus to fathom the seemingly irrational character of nature.

> Of the proportions holding between quantities, some strike me as being more perfect and others less so; the more perfect are those obtaining between proximate numbers, for instance, the double, triple, and sesquialter proportions, and so on; the less perfect are those obtaining between more remote prime numbers, such as the proportions 11 to 7, 17 to 13, 53 to 37, and so on; the imperfect finally are those obtaining between incommensurable [*incommensurabili*] quantities. These we can neither explain nor even name.
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> In these circumstances, if we had to organize and arrange to the best of our ability and in accordance with perfect proportions the differences between the principal motions of the celestial spheres, I believe that we should have to rely on proportions of the first type, which are the most rational; God, on the other hand, not bothering about symmetries [*simmetrie*] that man can understand, has ordered these motions with the help of proportions that are not only incommensurable [*incommensurabili*] and irrational but totally inaccessible to our intelligence...[40]

[40] M. Clavelin, *The Natural Philosophy of Galileo*, translated from French by A. J. Pomerans (Cambridge, MA, and London: MIT Press, [1968] 1974.), pp.447–8 (slightly modified). Galileo's letter of 16 July 1611 to Gallanzone Gallanzoni, is published in Favaro (ed.) [1890–1909] 1968, xi, 149–50: 'Ma io, per l' opposito, osservo, altre perfezioni essere intese dalla natura che noi intendere non possiamo, anzi pure che più presto per imperfezioni giudicheremmo: come, per essempio, delle proporzioni che cascano tra le quantità, alcune ci paiano più perfette, alcune meno; più perfette, quelle che tra i numeri più cogniti si ritrovano, come la dupla, la tripla, la sesquialtera, etc.: meno perfette quelle che cascano tra' numeri più lontani e contra sè primi, come di 11 a 7, 17 a 13, 53 a 37, etc.; imperfettissime, quelle delle quantità incommensurabili, da noi inesplicabili et innominate: talchè quando ad un huomo fusse toccato a dovere a sua elezione stabilire et ordinare con perfette proporzioni le differenze de i prestantissimi movimenti delle celesti sfere, credo che senza dubbio gl'haverebbe moderati secondo le prime et più rationali proporzioni; ma all'incontro Iddio, senza riguardo alcuno delle nostre intese simmetrie, gli ha ordinati con proporzioni non solamente incommensurabili et irrazionali, ma toatlmente impercettibili dal nostro intelletto…. Uno de i nostri più celebri architetti, se havesse hauto a compartire nella gran volta del cielo la moltitudine di tante stelle fisse, credo io che distribuite le haverebbe con bei

Galileo appears to be disturbed by the inability of humans to comprehend incommensurability in nature and for this reason he makes it a kind of divine 'mystery'.[41]

In sum, the attitudes of Kepler and Galileo are very different. For Kepler it is most likely that the planets have incommensurable periods. But for Galileo probability is not the issue, for the very possibility of an irrational element in nature (even in the mathematical sense) is unintelligible. Nevertheless, God can execute what is incomprehensible and inaccessible to ordinary human thought. Unlike Copernicus who had combined symmetry with harmony in a novel way, Kepler was concerned only with harmony. In order to present his discoveries coherently some *Spielraum* in nature had to be introduced. This was entirely consistent with Kepler's strong belief, in contrast to Galileo, that for the universe to be eternal it must accommodate incommensurable periods.

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partimenti di quadrati, esagoni et ottangoli, interzando le maggiori tra le mezzane et le piccole, con sue intese corrispondenze, parendogli in questo modo di valersi di belle proporzioni; ma all' incontro Iddio, quasi che con la mano del caso le habbia disseminate, pare a noi che senza regola, simmetria o eleganza alcuna le habbia sparpagliate.'

[41] On Galileo's view of reason and reality, see Clavelin [1968] 1974, ch. 8.